



Effect of Inclined Magnetic Field on the Peristaltic Flow of Non-Newtonian Fluid with Partial Slip and Couple Stress in a Symmetric Channel

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper focus on the variable inclined magnetic field on the peristaltic flow of non-Newtonian fluid. The angle between the magnetic and velocity fields is variable and all the flow variables are functions of two parameters coordinates and time. The equations which describing the diffusion of inclined magnetic field are derived and solved by using long wavelength approximation. We addressed some of the important physical concepts that have a direct relationship such as the pressure gradient, Pressure rise, velocity field, the stream line and trapping phenomena. This study is done through drawing many graphs by using the MATHEMATICA package.

Keywords: Couple stress; inclined magnetic field; non-Newtonian fluid; partial slip.

1. INTRODUCTION

Peristaltic problems have gained a considerable importance because of it applications in physiology, engineering, and industry. Such

applications include urine transport, swallowing food through the esophagus, movement of chime in the gastrointestinal tract, movement of ovum in the female fallopian tubes, vasomotor of small blood vessels, transport of slurries, corrosive

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fluids, sanitary fluids, and noxious fluids in nuclear industry. In view of these applications, several researchers investigated such fluid flow problem considering different subject like Noreen Sher Akbar et al. [1] have studied the Peristaltic flow of a Williamson fluid in an inclined asymmetric channel partial slip and heat transfer, Rama et al. [2] discussed the Second Law Analysis for Combined with Convection in Non-Newtonian Fluids over a Vertical Wedge Embedded in a Porous Medium, the results show that the Bejan number increases with the viscosity index and the buoyancy parameter, R. Ali et al. [3] analyzed Mixed convection heat and mass transfer of non-Newtonian fluids from a permeable surface embedded in a porous medium. N. Sandeep et al. [4] have shown the Effect of Inclined Magnetic Field on Unsteady free Convective flow of Dissipative Fluid past a Vertical Plate, N. Sandeep et al. [5] analyzed Radiation and inclined magnetic field effects on unsteady MHD convective flow past an impulsively moving vertical plate in a porous medium. N. Sandeep et al. [6] studied the Aligned magnetic field, radiation and rotation effects on unsteady hydro magnetic free convection flow past an impulsively moving vertical plate in a porous medium, Masoud A. Frand et al. [7] Study the Effect of Magnetic Field on Free Convection in Inclined Cylindrical Annulus Containing Molten Potassium. A. Malvandi et al. [8] analyzed MHD mixed convection in a vertical annulus filled with Al₂O₃-water nanofluid considering nano-particle migration, M. Modather et al. [9] studied MHD Mixed Convection Stagnation-Point Flow of a Viscoelastic Fluid towards a Stretching Sheet in

a Porous Medium with Heat Generation and Radiation, C.S.K. Raju et al. [10] analyzed Radiation, Inclined Magnetic field and Cross-Diffusion effects on flow over a stretching surface. C.S.K. Raju et al. [11] discussed the Effects of aligned magnetic field and radiation on the flow of Ferro fluids over a flat plate with non-uniform heat source/sink, C. Sulochana et al. [12] discussed the Radiation and Chemical Reaction Effects on MHD Nanofluid Flow over a Continuously Moving Surface in Porous Medium with Non-Uniform Heat Source/Sink.

With the above introduction the objective of this investigation is to analysis the effect of inclined magnetic field on the peristaltic flow of non-Newtonian fluid with partial slip and couple stress in a symmetric channel. The relevant equations are modeled and simplified using long wavelength approximation and the expressions for pressure rise has been calculated using numerical integration by software MATHEMATICA package, some of the important physical concepts that have a direct relationship like the pressure gradient, Pressure rise, velocity field, the stream line and trapping phenomena have been addressed.

2. MATHEMATICAL FORMULATION AND ANALYSIS

Consider MHD flow of an electrically conducting viscous fluid in asymmetric channel through porous medium. The lower wall of the channel is maintained at temperature T₁ while the upper wall has temperature T₀ as shown in the Fig. 1.

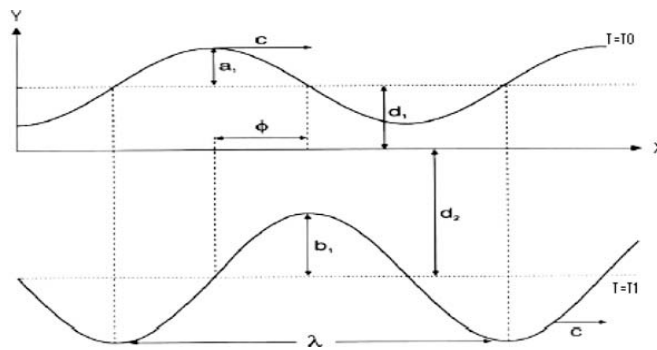


Fig. 1. Schematic diagram of a tow-dimensional asymmetric channel

The geometry of the wall surface is define as:

$$Y = H_1 = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X - ct)\right], Y = H_2 = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right] \quad (1)$$

Where a_1 and b_1 are amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, c is the velocity of propagation, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \leq \phi \leq \pi$, in which $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further

a_1, b_1, d_1, d_2 and ϕ satisfies the condition $a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2$. For the two dimensional incompressible flow, the governing equations of motion are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial X} + \nu \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\nu}{K} U - \frac{\sigma B_0^2}{\rho} * \cos \beta (U \cos \beta - V \sin \beta) - \frac{\eta}{\rho} \nabla^4 U \quad (2)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial Y} + \nu \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\nu}{K} V + \frac{\sigma B_0^2}{\rho} * \sin \beta (U \cos \beta - V \sin \beta)$$

Where $\nabla^4 = \frac{\partial^4}{\partial X^4} + \frac{\partial^4}{\partial Y^4} + 2 \frac{\partial^4}{\partial X^2 \partial Y^2}$

U, V are the velocities in X and Y directions in fixed frame, ρ is constant density, p is the pressure, ν is the kinematics viscosity, σ is the electrical conductivity, K is the permeability parameter, Introducing a wave frame (X, Y) moving with velocity c away from the fixed frame (x, y) by the transformation

$$x = X - ct, y = Y, u = U - c, v = V, p(x) = P(X, t) \quad (3)$$

Defining

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{d_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c}, \delta = \frac{d_1}{\lambda}, d = \frac{d_2}{d_1}, \bar{t} = \frac{ct}{\lambda}, h_1 = \frac{H_1}{d_1}, h_2 = \frac{H_2}{d_2}, \bar{p} = \frac{d_1^2 p}{\mu c \lambda}$$

$$a = \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, Re = \frac{cd_1}{\nu}, \bar{\psi} = \frac{\psi}{cd_1}, \bar{K} = \frac{K}{d_1^2}, M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1, \alpha = d_1 \sqrt{\frac{\mu}{\eta}} \quad (4)$$

Using the above non-dimensional quantities and neglecting the terms of order δ and higher, the resulting equations in terms of stream function $\psi (u = \frac{\partial \psi}{\partial y}, v = -\delta \frac{\partial \psi}{\partial x})$ can be written as:

$$\psi_{yyyy} - N^2 \psi_{yy} - \frac{1}{\alpha^2} \psi_{yyyyyy} = 0 \quad (5)$$

Since we are considering the partial slip on the wall, therefore, the corresponding boundary conditions for the present problem can be written as

$$\psi = \frac{q}{2} \quad \text{at} \quad y = h_1 = 1 + a \cos 2\pi x,$$

$$\psi = -\frac{q}{2} \quad \text{at} \quad y = h_2 = -d - b \cos(2\pi x + \phi),$$

$$\frac{\partial \psi}{\partial y} + L \frac{\partial^2 \psi}{\partial y^2} = -1 \quad \text{at} \quad y = h_1, \quad \frac{\partial \psi}{\partial y} - L \frac{\partial^2 \psi}{\partial y^2} = -1 \quad \text{at} \quad y = h_2,$$

The finishing couple stress boundary condition is:

$$\frac{\partial^3 \psi}{\partial y^3} = 0 \quad \text{at} \quad y = h_1, \quad \frac{\partial^4 \psi}{\partial y^4} = 0 \quad \text{at} \quad y = h_2.$$

Where q is the flux in the wave frame, a , b , ϕ and d satisfy the relation

$$a^2 + b^2 + 2ab \cos \phi \leq (1 + d)^2$$

The solution of the momentum equation straight forward can be written as

$$\psi = f_0 + f_1 y + f_2 \cosh m_1 y + f_3 \sinh m_1 y + f_4 \cosh m_2 y + f_5 \sinh m_2 y \quad (6)$$

Where

$$m_1 = \sqrt{\frac{\alpha^2 - \sqrt{\alpha^2(-4N^2 + \alpha^2)}}{2}}, m_2 = \sqrt{\frac{\alpha^2 + \sqrt{\alpha^2(-4N^2 + \alpha^2)}}{2}}, N^2 = \frac{1}{K} + M^2 \cos^2 \beta \quad (7)$$

the functions f_0, \dots, f_5 are large expressions will not mentioned here for sake of simplify. The flux and average volume flow rate is defined

$$Q = \frac{1}{T} \int_0^T \bar{Q} dt = \frac{1}{T} \int_0^T (q + h_1 - h_2) dt = q + 1 + d, \quad (8)$$

$$\frac{dp}{dx} = \psi_{yyy} - N^2 \psi_y - \frac{1}{\alpha^2} \psi_{yyyy} \quad (9)$$

$$\Delta P = \int_0^1 \frac{dp}{dx} dx \quad (10)$$

The axial velocity component in the fixed frame is given as

$$U(X, Y, t) = 1 + \psi_y = 1 + f_1 + f_2 m_1 \sinh m_1 y + f_3 m_1 \cosh m_1 y + f_4 m_2 \sinh m_2 y + f_5 m_2 \cosh m_2 y \quad (11)$$

Where $h_1 = 1 + a \cos[2\pi(X - t)]$ and $h_2 = -d - b \cos[(2\pi(X - t) + \phi)]$

3. RESULTS AND DISCUSSION

In this part, we discussed the results by the graphical illustrations for different physical quantities. Figs. 2-7 show the pressure gradient for different value of Permeability parameter K , couple stress α , magnetic field M , inclined

magnetic field $\cos^2 \beta$, amplitude ratio ϕ , partial slip L . We find that pressure gradient is maximum at $X=0.5$ for $\alpha=1.5$, $K=.1$, $M=4$, $L=.1$, $\cos^2 \beta = 10$, the pressure gradient increase when the parameters M and $\cos^2 \beta$ increases.

As shown in Fig. 4 and Fig. 7, the pressure gradient decreases when the other parameters increase.

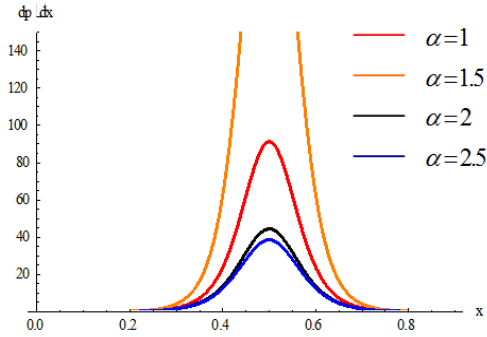


Fig. 2. Variation of dp/dx with x for different values of α at , $k=1000, M=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, \phi =0.001, \cos^2 \beta =.5$

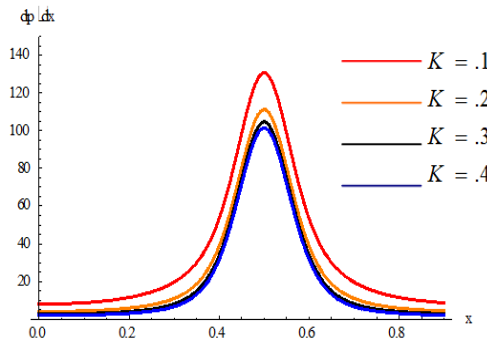


Fig. 3. Variation of dp/dx with x for different values of K at , $\alpha=1, M=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, \phi =0.001, \cos^2 \beta = 10$

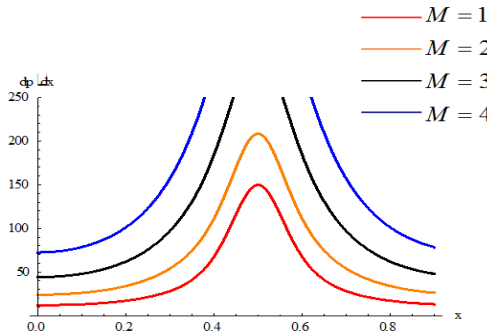


Fig. 4. Variation of dp/dx with x for different values of M at $\alpha=1, K=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, \phi =0.001, \cos^2 \beta = 5$

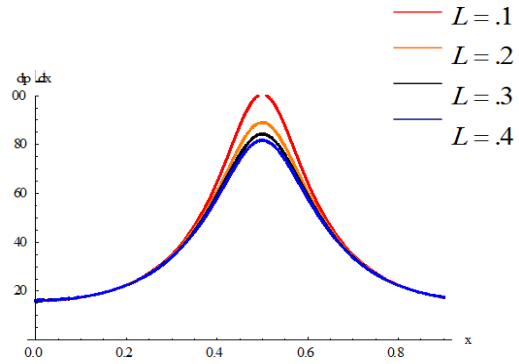


Fig. 5. Variation of dp/dx with x for different values of L at $\alpha=1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, M=1, \phi =0.001, \cos^2 \beta =10$

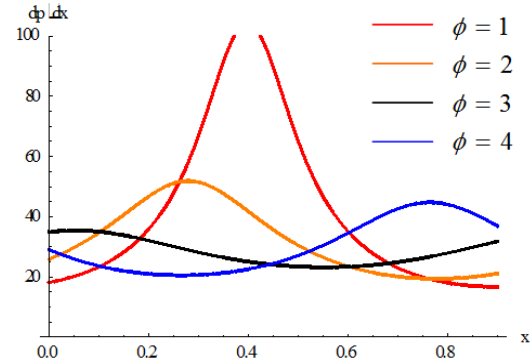


Fig. 6. Variation of dp/dx with x for different values of ϕ at $\alpha=1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, M=1, L =0, \cos^2 \beta =10$

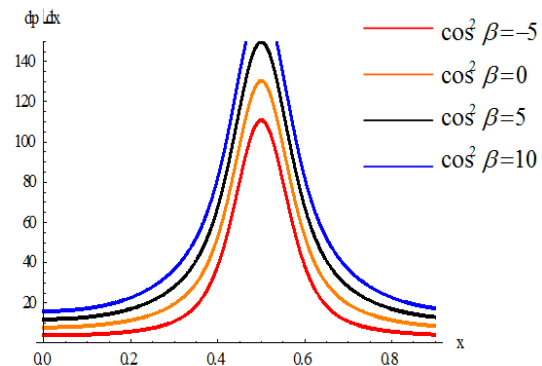


Fig. 7. Variation of dp/dx with x for different values of $\cos^2 \beta$ at $\alpha=1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, M=1, L =0, \phi =.001$

Pressure rise is important physical measures in peristaltic mechanism , so Figs. 8-13 shows the effect of $L, \alpha, M, K, \phi, \cos^2 \beta$, on pressure rise We found that increases in α, L, K , the Δp increases and increases in $M, \cos^2 \beta, \phi$ the Δp will be decreases. In Fig. 8 Δp increases for $Q < -0.3$ and for $Q > -0.3$, Δp has an opposite behavior. In Fig. 9 Δp decreases for $Q < -0.4$ and for $Q > -0.4$, Δp has an opposite behavior.

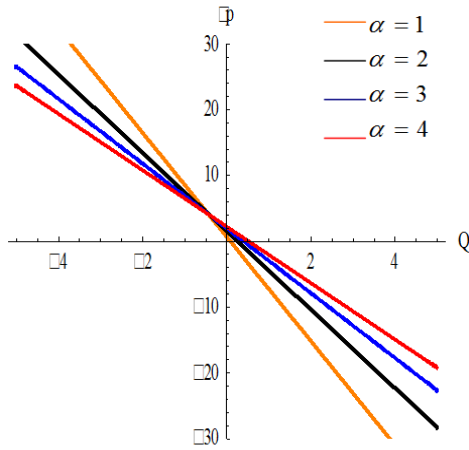


Fig. 8. Variation of Q with Δp for different values of α at $\phi = \frac{\pi}{6}, k=.2, M=2, d=2, a=0.7, b=1.2, L=0.02, y=1.4, \cos^2 \beta =10$

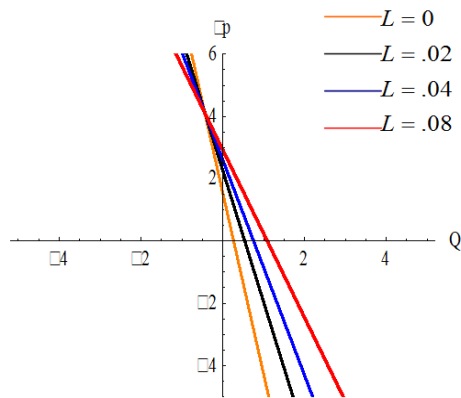


Fig. 9. Variation of Q with Δp for different values of L at $\alpha = 2, \phi = \frac{\pi}{6}, k=.1000, M=2, d=2, a=0.7, b=1.2, y=1.4, \cos^2 \beta =10$

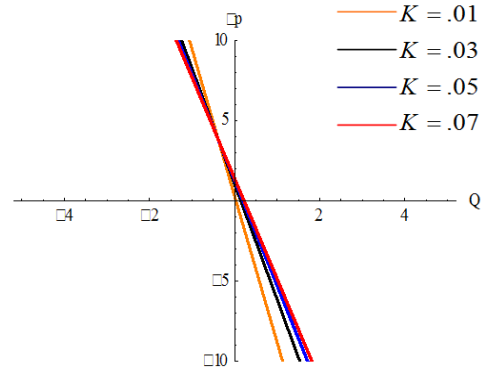


Fig. 10. Variation of Q with Δp for different values of K at $\alpha = 2, M=2, d=2, a=0.7, b=1.2, L=0.04, \phi = \frac{\pi}{6}, y=1.4, \cos^2 \beta =10$

Fig. 10 Δp decreases for $Q < -0.3$ and for $Q > -0.3$, Δp has an opposite behavior. Fig. 11 Δp increases for $Q < 0.3$ and for $Q > 0.3$, Δp has an opposite behavior, in Fig. 12 Δp increases for $Q < 2.5$ and for $Q > 2.5$, Δp has an opposite behavior.

Figs. 14-19 illustrate the velocity field for different values of $\alpha, K, M, \cos^2 \beta, \phi$ and L. It is observed that the velocity field increase when K increase and velocity field decrease when the other parameters increase and finally the shapes looks like a parabola and it can be noticed that the velocity take the maximum value in the middle.

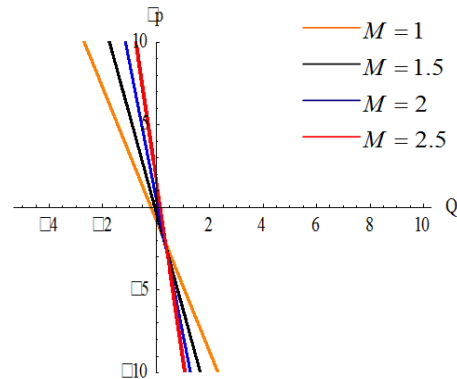


Fig. 11. Variation of Q with Δp for different values of M at $\alpha = 1, K=10, d=2, a=0.7, b=1.2, L=0.04, \phi = \frac{\pi}{6}, y=1.4, \cos^2 \beta =10$

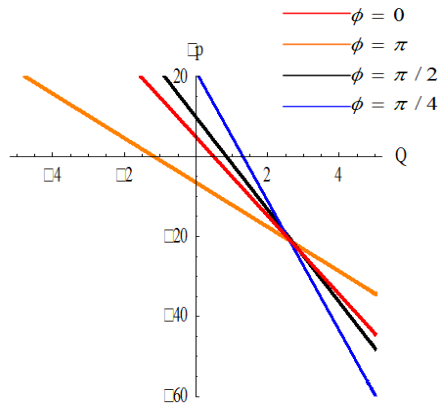


Fig. 12. Variation of Q with Δp for different values of ϕ at $\alpha = 1, M=2, d=2, a=0.7, b=1.2, L=0.04, y=1.4, \cos^2 \beta = 10$

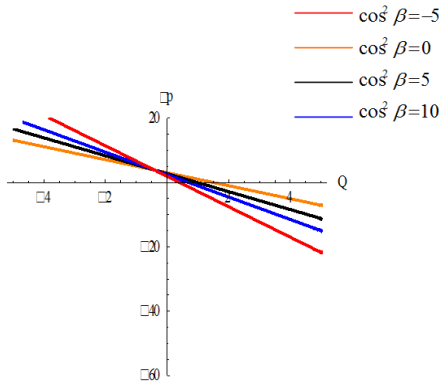


Fig. 13. Variation of Q with Δp for different values of $\cos^2 \beta$ at $\alpha = 1, K=.1, d=2, a=0.7, b=1.2, L=0.04, \phi = \frac{\pi}{6}, y=1.4$

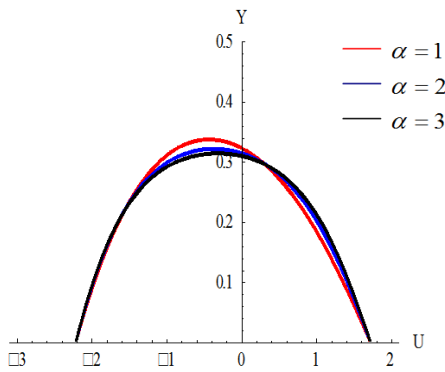


Fig. 14. The velocity at $Q=-1, a=0.7, b=1.2, \phi = 0, x=1, k=2, M=1, d=1, L=0.02, \cos^2 \beta = 5$

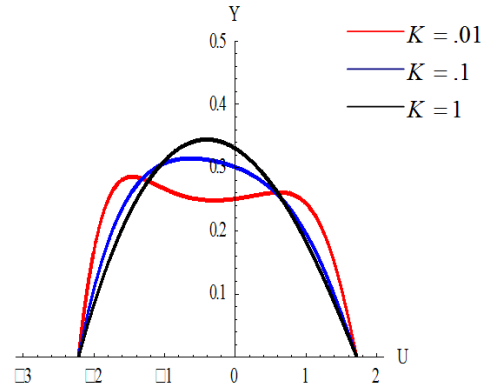


Fig. 15. The velocity at $\alpha=1, M=1, d=1, Q=-1, a=0.7, b=1.2, L=0.02, \phi = 0, x=1, \cos^2 \beta = 1$

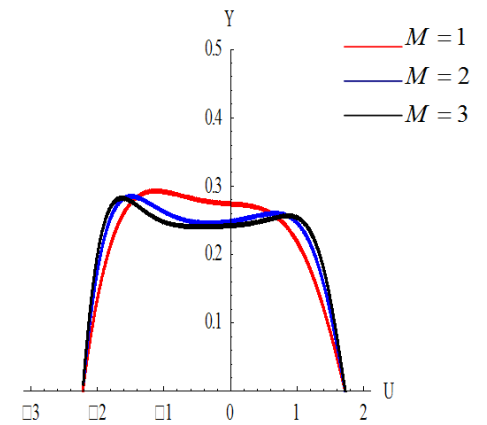


Fig. 16. The velocity at $\alpha=1, k=.1, d=1, Q=-1, a=0.7, b=1.2, L=0.02, \phi = 0, x=1, \cos^2 \beta = 3$

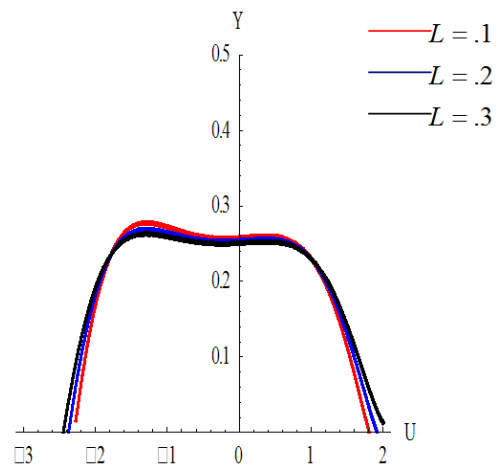


Fig. 17. The velocity at $\alpha=1, k=.1, M=1, d=1, Q=-1, a=0.7, b=1.2, \phi = 0, x=1, \cos^2 \beta = 5$

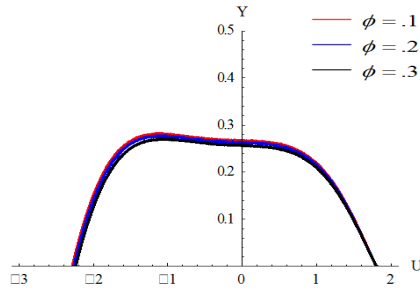


Fig. 18. The velocity at $\alpha=1, k=1, M=1, d=1, Q=-1, a=0.7, b=1.2, L=0.02, x=1, \cos^2 \beta = 3$

Finally, the formation of an internally circulating bolus of streamline is called trapping and this trapped bolus pulled ahead along with the peristaltic wave so Figs. 20-25 describe the stream line and trapping phenomena for different value of $\alpha, L, M, Q, \cos^2 \beta$ and ϕ . We noticed that all diagram were not symmetric. The trapping for different values of M, l and $\cos^2 \beta$

are shown in Figs. 20, 21 and 25. It is seen that trapped bolus and stream line decreases in size as M, l and $\cos^2 \beta$ increase and slowly disappear for the large value, while in Figs. 22 and 24 increase of the parameters q and α the trapping and stream line will be increase.

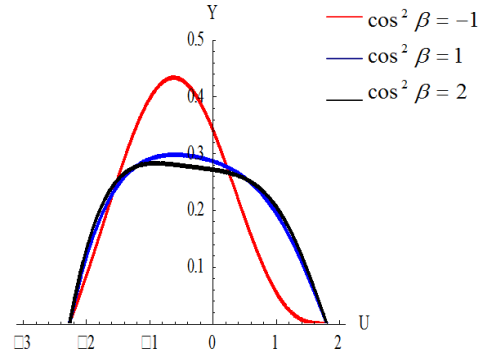


Fig. 19. The velocity at $\alpha=1, k=1, M=1, d=1, Q=-1, \phi = 0, a=0.7, b=1.2, L=0.02, x=1$

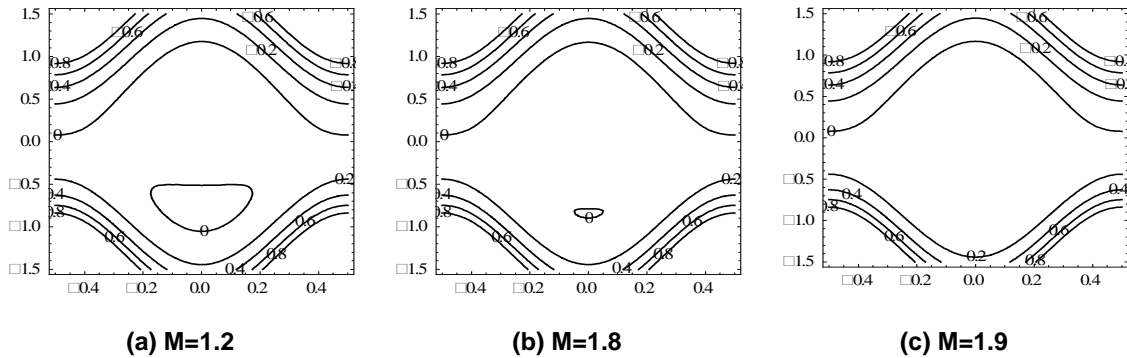


Fig. 20. Stream line for different values of M , and $\alpha=1, k=0.2, d=1, Q=1.5, a=0.5, b=0.5, L=0.02, \phi = 0, \cos^2 \beta = 1$

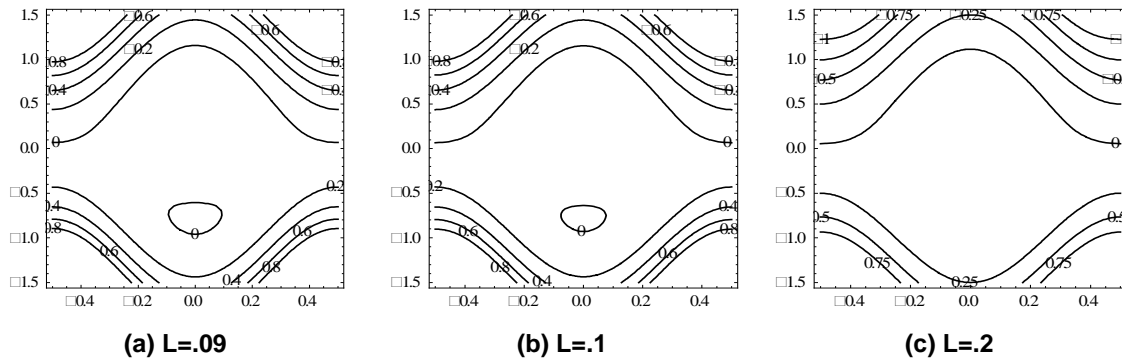


Fig. 21. Stream line for different values of L , and $\alpha=1, k=0.2, d=1, Q=1.5, a=0.5, b=0.5, \phi = 0, M=1, \cos^2 \beta = 1$

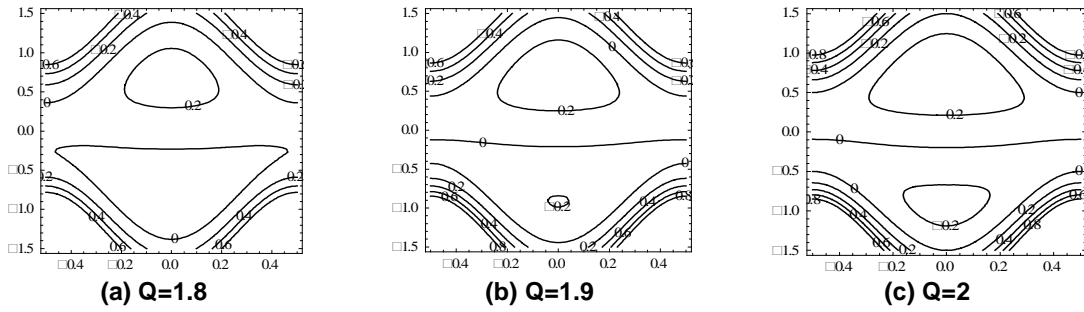


Fig. 22. Stream line for different values of Q , and $\alpha=1, k=0.2, d=1, a=0.5, b=0.5, \phi=0, M=2, L=0.02, \cos^2 \beta=1$

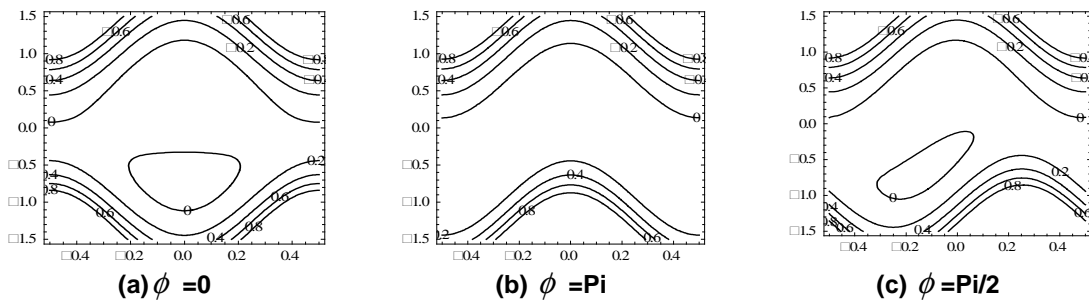


Fig. 23. Stream line for different values of ϕ , and $\alpha=1, k=0.2, d=1, Q=1.5, a=0.5, b=0.5, L=0.02, M=1, \cos^2 \beta=1$

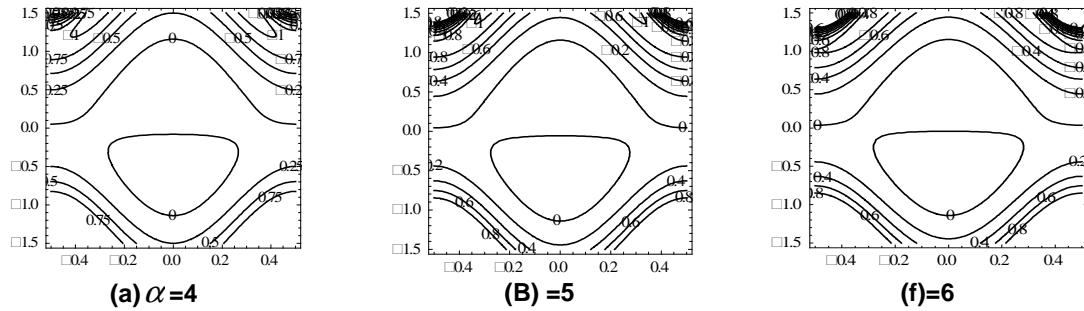


Fig. 24. Stream line for different values of α , and $k=0.2, d=1, Q=1.5, a=0.5, b=0.5, L=0.02, M=1, \cos^2 \beta=1$

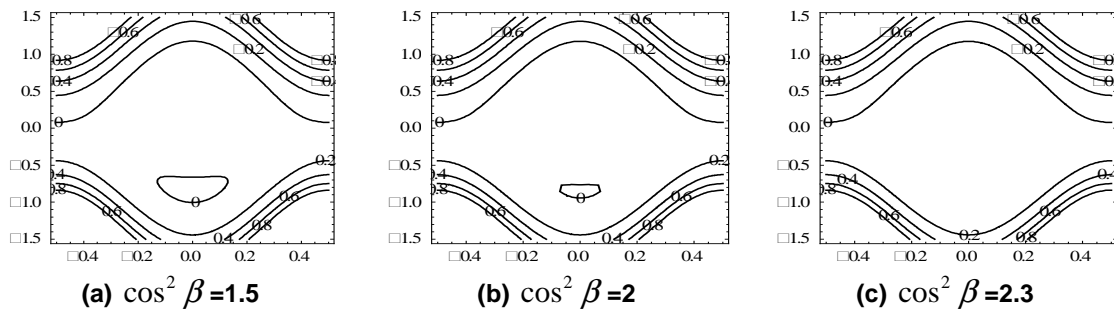


Fig. 25. Stream line for different values of $\cos^2 \beta$, and $\alpha=.5, k=0.2, d=1, Q=1.5, a=0.5, b=0.5, L=0.02, M=2$

4. CONCLUSIONS

We have discussed the results with inclined magnetic field on the peristaltic flow of a non-Newtonian fluid with couple stress and partial slip. The results are discussed through graphs. We have concluded the following observations:

1. The pressure gradient decreases with the increases in M , $\cos^2 \beta$.
2. The pressure rise decreases with the increases in L and increases when α, M , $\cos^2 \beta$ and K
3. The axial velocity increases with the increase in k . Further, the axial velocity decreases with the increase in M , $\cos^2 \beta$, α , L , ϕ .
4. All diagram which refers to stream line were not symmetric
5. The size of the trapped bolus and stream line decreases by increasing in M, L and $\cos^2 \beta$.
6. The size of the trapped bolus and stream line increases by increasing Q , α .

COMPETING INTERESTS

Author has declared that no competing interests exist.

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