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Using Simulation as a Problem Solving Method in Dice Problems

Timur Koparan[1*]

[1]Bülent Ecevit University, Zonguldak, Turkey.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This study aims to develop a learning environment for teaching probability by using dynamic software, and investigate its efficiency. The sampling of the study includes 144 prospective mathematics teachers from Bülent Ecevit University. The study utilizes case study methodology. The data were gathered through a data collection tool which consisted of open-ended questions, and observations. Prospective teachers (PTs) were, first, asked to answer the open-ended questions by using pen and paper. The answers were classified and analysed. Analyses showed that the responses, in general, were not consistent. The findings reveal that PTs had difficulty in making decisions and gave mostly inconsistent responses before they used a simulation. In simulation utilization process, they had some opportunities such as defining a trial number, classifying data, displaying in graphs, and making generalisations.

Keywords: Statistics education; prospective teachers; using simulation; probability; problem solving; dice problems.

**Corresponding author: E-mail: timurkoparan@gmail.com;*

1. INTRODUCTION

Probability occupies the top of the list of subjects in which teachers have difficulty teaching and students learning [1,2,3,4,5,6,7,8]. [9] argued that probability education could not be reduced only to conceptual structures and problem solving tools, and made emphasis, at the same time, on the ways of logical inferences and the obligation of creating proper intuitions for students. [10] claimed that people are inclined to believe that the same cause will produce the same effect. Such conceptual mistakes in the process of learning probability concepts may influence the important personal decisions concerning daily life [11]. Although probability is a socially useful and important branch in mathematics education, many challenges arise because of the discrepancies between intuitions and truths [8]. One of the causes of these difficulties is limited experience in general. Preparatory process both the teachers and students spend in learning statistics and probability is inadequate. Teacher applications in this subject are usually within the boundaries of course books and not beyond some calculations. Students have great difficulties in comprehending abstract probability concepts [5]. That the concepts are left abstract in a subject that is mostly taught through conventional methods poses a real problem. These concepts should be concretized to increase the attitude and interest of the students.

Since the necessary visuals to study some probability problems are not met in conventional mediums, alternative learning environments are needed [8]. Modern teaching methods, which encourage students to be more active, should now substitute for traditional teaching methods [5]. Suggestions in this field are to create awareness in students about probability structures and applications, and make use of technology for data analysis and improvement of conceptual understanding [12,13]. Some researchers on teaching probability have offered use of computers as a way of understanding abstract or difficult concepts and supporting the proliferation of student capabilities [3,5,14].

To use different software as a tool is inevitable in this period, which necessarily involves the inclusion of new teaching technologies in learning environments. The software has increasingly developed and made it quick and easy to observe some problematic situations, which are not possible to gain in real life. One of these is the simulation software. Simulation software is a teaching method in which learners can change the parameters and make experiments personally [15]. Simulations provide opportunities to enhance students' conception of statistical ideas [16] and support the learning process of those who work on chance experiments [17,18]. Simulation-based activities enable students to build up their own knowledge [19]. In addition, simulations encourage, in the learning process, active learning and participation. They can be used as tools in developing conceptual understanding and problem solving. In this way, students might, by trying more alternatives, play a more active role in finding answers and solutions to their own questions and problems [20]. Rather than close-ended questions, offering simulation and design activities with no clear answers but chances to discover might help students improve some skills necessary for lifelong learning. [21] point out that the students can run simulations in computer classes at schools to solve some simple probability problems that are not possible in physical experiments. Combined with the use of technology, simulation is the best strategy to focus on concepts and reduce technical calculations [11].

Knowing the difficulties in teaching probability and making efforts to overcome them are very important. In this study PT's interest in technology was used to overcome the difficulties in teaching possibilities. This study, while mainly examining the use of simulations in probability teaching, presents the strategies used by PTs in solving probability problems, necessary content and examples of use of technology in probability teaching. The application is also an example as to how technology can be integrated into mathematics lessons. The study is expected to be useful to both researchers and teachers.

1.1 TinkerPlots Dynamic Statistics Software and Sampler Toolbar

Used by middle school students through to university students, TinkerPlots is software which offers a dynamic learning environment with data analysis and modelling tools [22]. It is deemed to be an important and useful tool in the development of ideas about experimental probability. As the number of trials can be changed, it provides great opportunities to observe big and small samples. Data gained through trials are converted with tables and charts into a dynamic and visual study medium. Thus, thanks to the activities, students get to improve an understanding of concepts related to experimental probability rather than theoretical probability, and have the chance to verify or change their intuitions about probability and randomness.

1.2 Theoretical Framework

The activities in this study are built on modelling, simulations and "basic logic of inference" which was laid down by [23]. This logic of Cobb is about random experiments and random samples and he named it as 3R (randomize-repeat-reject). In this respect, the steps below were followed.

1. Forming a special model that includes reasonable change in the outcomes attributed to random processes;
2. Using the model to produce simulation data; and
3. Evaluating the distribution of the data produced by the simulation.

1.3 Purpose and Research Problem

This study aims to develop a learning environment for probability teaching by using dynamic software and study the efficiency of this learning environment, and evaluate the effectiveness of using simulation as a problem solving method. This study will also try to find answers to questions of how simulation can be used as a problem solving method, and how technology can be integrated to mathematics teaching. Accordingly, the PTs were asked to answer three questions related to an experiment on rolling two dice first by using pen and paper, and later compare their responses with the results of the simulations prepared for these problems. For this purpose, the research problem is as follows: What is the effectiveness on PT's thinking of using simulation as a problem solving method?

2. THE STUDY

Case study method was used for this study. The most significant feature of case studies is that they shed light on a phenomenon by scrutinizing one single case that belongs to the phenomenon [24]. In these studies, medium, individuals and processes are researched holistically and the roles and relationships in the process are the focus. More than one data collection technique could be used in these studies and it is possible to have data variety, which can enrich and support additional data sources [25]. This study aims to inspect simulation use as a special case to solve real life problems. This particular application means to define thinking ways of PTs in different learning environments as they investigate real life problems.

2.1 Sampling

This study was carried out at Bülent Ecevit University in the 2014-2015 Academic Year Fall Semester. The sample of the study is comprised of 144 PTs who took the course "Teaching Technologies and Material Design". Prospective mathematics teachers were from 3 different classes and they were in their final year of undergraduate education. They first met during this study and have used the software. Eighty-three prospective teachers were female and 61 male.

2.2 Data Collection Tools and Data Analysis

The study employs, as a data collection tool, a test that is comprised of three open-ended questions and TinkerPlots dynamic statistics software. The open-ended questions were developed by the researcher with respect to such points as being applicable to real life and being able to be modelled. The problems in the data collection tool are displayed in Table 1.

The responses from the PTs to the open-ended questions were analysed qualitatively and quantitatively. They were read again and again. Different answers from each question were classified (consistent, inconsistent and no answer), frequency was calculated, and at the same time percentage rates were calculated. Also typical responses were presented as qualitative. The same questions were asked after answer using simulations. Use of simulations and changes in the way of thinking were observed. By taking the phases of creation, utilization and evaluation of the model into account, data from the simulations were presented with screenshots.

2.3 Operation

In the first stage of the study, PTs were asked to respond to the real life problems related to the roll of a pair of dice. By analysing the data gained, their ways of thinking were generalized as trends. In the second stage, "Sampler" toolbar of TinkerPlots was introduced and simulation activities were performed for coin tossing and dice rolling experiments. Later, PTs were assigned, in accordance with the nature of the problems, to form, use, and evaluate the simulations. The activities were executed within the scope of the course "Teaching Technologies and Material Design", and took 4 hours a week and 8 hours in total. The PTs had had no experience of TinkerPlots until this study. During the activities, the researcher made participant observations.

2.4 Limitations and Assumptions

1. Research is limited to 3 questions and models about the experimental possibilities.
2. The use of simulation is limited to TinkerPlots Sampler tool.
3. Modeling studies is limited to 2 weeks (total 8 hours).
4. It is assumed that the questions answered by the participants provided honest and sincere data.

3. FINDINGS

The findings from the three open-ended questions and simulations were analysed together. Responses from pen and paper were classified, and frequency and percentage rates were calculated. While thinking ways are presented under different themes, the data from simulations are shown in screenshots.

3.1 Findings from the 1st Problem

In the first problem, PTs were asked to elucidate their predictions with reasons to the question how many times each side appears when a die is thrown into the air. Distribution of the responses gained from this problem is shown in Fig. 1.

As it could be viewed in Fig. 1, 16.67% of the responses are consistent, 69.44% inconsistent, and 13.89% had no answers. Some of the consistent responses from PTs to the problem are given below.

Consistent responses take variance into account and, at the same time, are comprised of data sets sums of 60. For example;

- *"10,8,7,5,15,15" and "15,10,13,8,5,9" etc.*

Inconsistent responses include no or too much variance, and consist of data sets sums, which are not 60. For example;

- *"1,21,18,7,5" and "17,15,13,7,7,1" etc.*
- *"Each side appears two times."*
- *"Of a dice 60 times rolled, all might turn up 1."*

Some of PTs were seen to be firmly dependent on theoretical probability. In this experiment of 60-times-rolled-dice, for instance, some of the PTs stated that each side would appear 10 times. However reasonable it seems in the theoretical perspective, this is rarely the case in experimental studies. Therefore, such answers were counted in the inconsistent responses. Below are shown some examples of such answers.

- *"Each side will appear 10 times because it has six sides."*
- $A = \{1,2,3,4,5,6\}$, $s(A) = 6$ *Since the number of elements of sample space in the experiment of rolling two dice is* $s(E) = 36. P(A) = \frac{6}{36} = \frac{1}{6}$ *probability of each side to appear is equal.*

In addition to these, some of the PTs gave irrelevant (idiosyncratic) answers.

- *"When we roll a dice it might turn up heads or tails. I think they have equal chances".*
- *"A pair of dice is thrown into air but what sort of ground does it fall to? This is not mentioned".*
- *"It appears maximum 60, minimum 0 times".*

Table 1. The problems in the data collection tool

Problem no.	Problem sentence
1	When a die thrown into air 60 times, how many times does each side appear? Explain your prediction and its reason.
2	Think about the sum of the numbers that turn up when a pair of dice is rolled. When 1000 trials are made in this way, which result will be more likely than the others? Predict and explain your reason.
3	When a pair of dice is rolled, is the sum 11 or 12 more likely? Or are both sums equally likely? Predict and explain your reason.

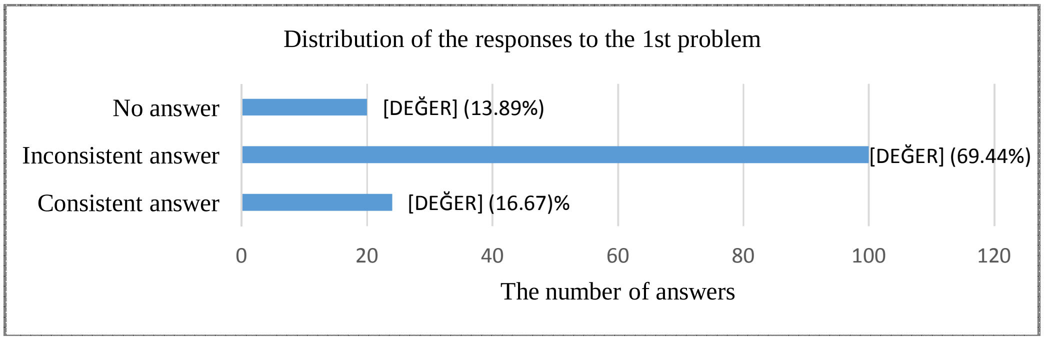


Fig. 1. Distribution of the responses to the 1st problem

It was observed that, in most of the idiosyncratic responses, PTs (17 PTs) confused coin tossing with the experiment of rolling dice. Simulations from PTs for the first problem are given below.

Model: PTs formed the model seen in above Fig. 2 by taking into account that there is a set of dice (Draw 1), and it is used for 60 trials (Repeat 60). Thus, the model is ready to work.

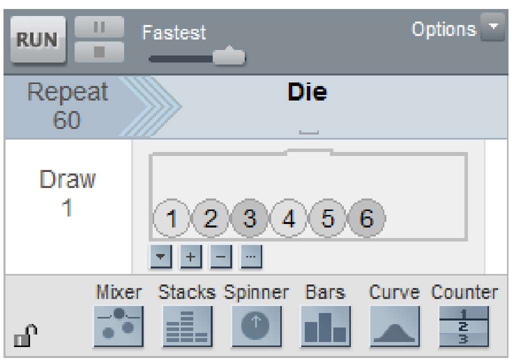


Fig. 2. Simulation for the1st problem

Using the model: The model developed by PTs was used for the experiment of rolling dice 60 times. Results were recorded. The screenshot on the left in Fig. 3 displays the results gained following 60 rolls of the dice.

Evaluation: The simulation in Fig. 2 was used by PTs for 60 trials. Results that appear on the left in Fig. 3 were gained. These results were classified by TinkerPlots and the chart on the right in Fig. 3 was drawn. The chart, which shows how many times each face turned up, gave an idea to the PTs about the answer of this open-ended question and helped them to make an evaluation and decision easily. Some oral reviews are as follows.

- *"I thought it would be equal for all faces but it wasn't. In none of my trials was there an equal number of distribution".*
- *"It could be less than 10. It could be more than 10. Each face of die appears nearly 10 times".*
- *"I thought only one face would turn up in all my 60 trials. However, in all my trials, it wasn't the case that only one face would turn up or any of the faces wouldn't appear".*

PTs reported that they, before the simulation, had difficulty reasoning to find a solution and couldn't be sure of their decisions. However, following the use of the simulation, they had a clear solution of the problem.

3.2 Findings from the 2nd Problem

In the second problem, PTs were asked to think about the sum of the number that would turn up when a pair of dice was thrown and which result would outdo the others in 1000 trials made in this way, and explain their predictions with reasons. The distribution of the responses to this question is displayed in Fig. 4.

As it could be seen in Fig. 4, 43.75% of the responses were consistent, 45.14% were inconsistent and 11.11% had no answers. Some examples of consistent answers from PTs are shown below.

- *"It is more likely that the sum is 7 because you can have 7 more than others with the numbers on the dice."*
- *"The sum is more likely to be 6, 7 and 8."*
- *"1+1=2, 1+2=3, 2+1=3, 2+2=4, 1+3=4, 3+1=4, 1+4=5, 4+1=5, 2+3=5, 3+2=5, 1+5=6, 5+1=6, 2+4=6, 4+2=6, 3+3=6, 1+6=7, 6+1=7, 2+5=7, 5+2=7, 3+4=7,*

4+3=7, 2+6=8, 6+2=8, 3+5=8, 5+3=8, 4+4=8, 3+6=9, 6+3=9, 4+5=9, 5+4=9, 4+6=10, 6+4=10, 5+5=10, 5+6=11, 6+5=11, 6+6=12" There is sum 2 in 1 situation, sum 3 in 2 situations, sum 4 in 3 situations, sum 5 in 4 situations, sum 6 in 5 situations, sum 7 in 6 situations, sum 8 in 5 situations, sum 9 in 4 situations, sum 10 in 3 situations, sum 11 in 2 situations and sum 12 in 1 situations. As it is clearly seen, the situations in which the sum is 7 appear more than others so in 1000 trials sum 7 will happen more often than other sums.

Some inconsistent answers from PTs are given below.

- *"Probability of all numbers except 1 is equal."*
- *"Arithmetic mean of the numbers is 3.5. If we multiply it with 1000 it makes 3500."*
- *"That the sum is 6 is more likely."*
- *"Numbers, sum of which are even, are more likely to appear."*
- *"5 will turn up more."*
- *"It is more likely that the sum is 8."*
- *"It is more likely that the sum is 11."*
- *"Sum 2 will appear more."*
- *"Most 12, most 6, most 8, most 10, most 5."*
- *"6 will turn up most. Because if 1 appears 6 times it will be equal 6 that appears once."*
- *"If the least is 1-1, greatest will be 6-6. When thrown 1000 times it will be between 2000 and 12000."*
- *"Possibility to turn up is equal for all. 1/6.1/6=1/36"*
- *"Sum will vary between 2 and 12."*

Some inconsistent answers are observed to be made up of idiosyncratic responses and personal experiences. For example;

- *"3 will appear most. Because the dice I throw turn up 1 or 2."*
- *"It will be 7 because 7 is my lucky number."*

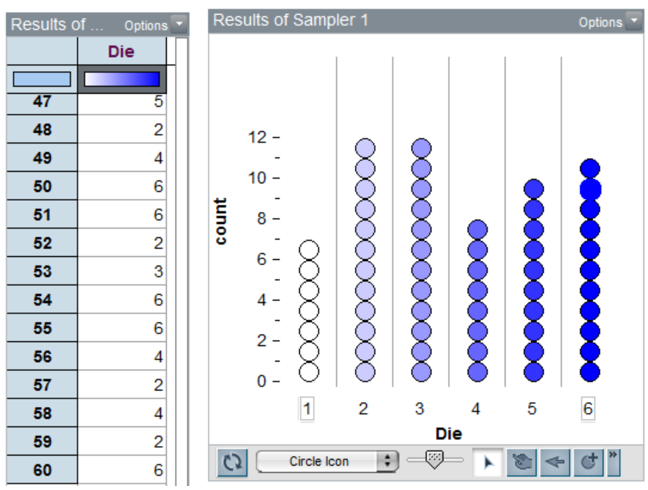


Fig. 3. Data production with the simulation of the 1st problem

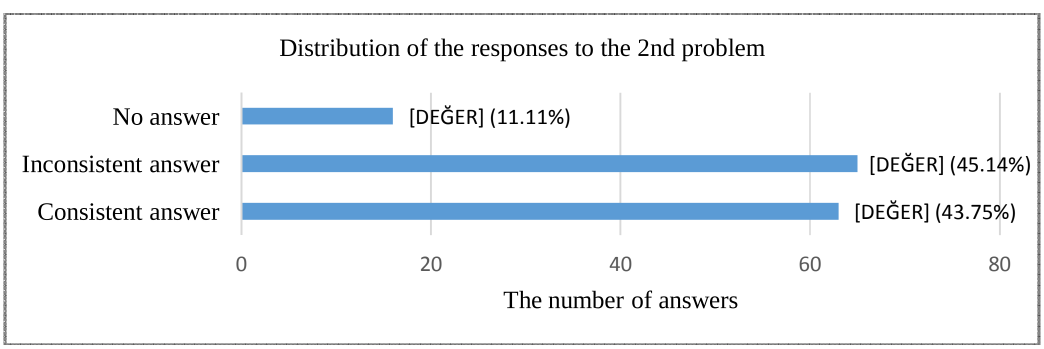


Fig. 4. Distribution of the responses to 2nd problem

Some PTs didn't answer the question or wrote that it wasn't possible to make a prediction.

- *"Difficult to predict. It is uncertain."*
- *"It totally depends on luck. Probability of winning the lottery in New Year's raffle is 50%. You win it or not. We can't know that."*

As it is clear in the responses, PTs made very different explanations, which show that most of them were in confusion in this question. Simulation studies from PTs to the 2nd problem are given below.

Model: Taking into consideration that there are 2 dice (Draw 2) and 1000 trials to be made (Repeat 1000), they formed the model displayed in Fig. 5. Thus, the model is ready to work.

Fig. 5. Simulation for 2nd problem

Using the model: The model in Fig. 5 was formed, tried, and controlled by PTs. Data gained from 1000 trials is displayed on the left in Fig. 6.

Evaluation: Although the simulations for the second problem were the same, the data gained from the simulations were not 100% the same since simulations produce different data in each trial. As a consequence, the general results are not the same. The data for 1000 trials in the table as shown in Fig. 6 were, with the help of software's graphical features, used to produce the graph on the right in Fig. 6. This graph shows the distribution of the results for 1000 trials. Following the formation of the graph, all of the PTs were observed to give a more consistent account of the problem. Some oral reviews are as follows.

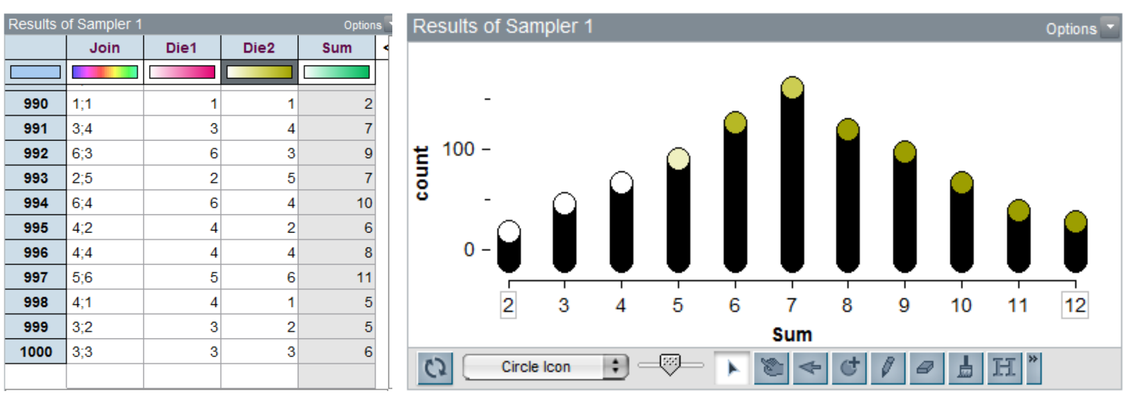
- *"The case that the sum is 7 is more frequent."*
- *"Sums of numbers from the dice show normal distribution."*
- *"I would never think that this would happen. After thinking why this happened, I realised the mathematical truth behind it."*

3.3 Findings from the 3rd Problem

In the third question, PTs were asked to make a prediction and explanation about when a pair of dice was rolled, whether the sum 11 or 12 is more likely or both have equal chances of occuring. Distribution of the responses to this question is shown in Fig. 7.

As Fig. 7 shows, of the responses from PTs to 3rd problem, 14.58% was consistent, 82.64% was inconsistent, and 2.78% had no answers. An example of a consistent one is given below.

- *"For the sum 11, you need 5-6 or 6-5. As for 12, you need 6-6. There are two instances for 11 while there is only one for 12. Sum 11 is more likely."*



Results of Sampler 1

	Join	Die1	Die2	Sum
990	1;1	1	1	2
991	3;4	3	4	7
992	6;3	6	3	9
993	2;5	2	5	7
994	6;4	6	4	10
995	4;2	4	2	6
996	4;4	4	4	8
997	5;6	5	6	11
998	4;1	4	1	5
999	3;2	3	2	5
1000	3;3	3	3	6

Fig. 6. Data production and evaluation with simulation for 2nd problem

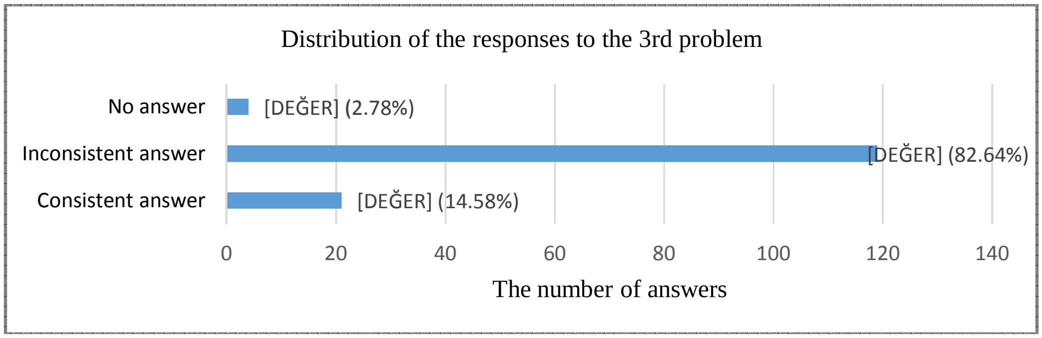


Fig. 7. Distribution of the responses to 3rd problem

Some examples of inconsistent answers are as follow.

- *"These two are equal."*
- *"Since they are independent events, they have equal chances to happen."*
- *"12 is more likely."*
- *"When two dice rolled, their probability to turn up the same is more likely."*
- *"11 is odd, 12 is even. Even numbers have many divisors. When we add these, it's more probable to have 12."*
- *"Only chance for 11 is 5-6 and for 12 it is 6-6, there is only one instance for both of them."*
- *"Equal, there are equal numbers in each dice. 11=6+5 probability for each is 1/6, 12=6+6 probability for each is1/6."*
- *"Two dice are independent from each other. I think these are equal, no difference."*
- *"Their probability to turn up is the same. They either do or don't."*

Some inconsistent answers are idiosyncratic and from personal experiences.

- *"Small numbers are easier to get."*
- *"It is difficult to throw 6-6 in ludo."*
- *"It is more difficult to get 6-6. I think 5 appears more when you roll dice".*

Some PTs didn't answer the question or said that a prediction can't be made.

- *"It depends on luck."*
- *"It can't be predicted."*

It was not required to form a different model for the 3rd problem. The model formed for the 2nd problem in Fig. 5 also includes the solution of this problem. When the data produced with the use of the simulation displayed in Fig. 5 are examined, it is also understood from the graph in Fig. 6 that, in the experiment of two dice rolled, it is more probable to have 11 than 12 from the faces that turn up. After the graph in Fig. 6 was gained, it was observed that all of the PTs realised the more likely case and, discarding their misconceptions, came to face the truth of the situation. Some oral reviews are as follow.

- *"With fewer trials, it wasn't possible to notice it. It is realised better with the increased number of trials. For example, sum 11 always happened more than sum 12 for 1000 trials".*
- *"I thought 12 would be more since it is even. However, in my trials in the simulation I found 11 to happen more".*

It could be understood from these answers that simulations give ideas about general truths for the problems and provide the PTs who think differently with opportunities to re-think. Moreover, they help them make connections between real cases and mathematical concepts and relationships.

4. DISCUSSION AND CONCLUSIONS

Since the necessary visuals to study probability problems cannot be delivered in traditional mediums, alternative learning environments are needed. In this sense, simulations with their different features, and create appropriate learning environments for both teachers and students. [26] pointed out that using various teaching materials provides students with the opportunity not only to organize and structure their own knowledge but also to deal with mathematical concepts and structures in different perspectives. Correspondingly, [27] argued that materials and visual examples improve students' abilities to organize their knowledge so that they transfer this knowledge into practice and use it to solve problems in a clever and successful way.

This study aimed to examine the efficiency of using simulations in PTs' decision-making processes. Within this scope, PTs were asked three open-ended real-life questions that are convenient to make simulations. [28] proposed, through observation, simplifying a real-life problem, forming a model and carrying out mathematical studies with this model and assessing the results had conceptual value. It was ascertained in this study that, before simulations were used, a majority of the responses from the PTs were inconsistent and, following the simulations, all of the responses were consistent. It was observed that the problems were responded to in a way that disregarded variance and included only theoretical probability or was just idiosyncratic, and very few of these are consistent with mostly no supporting explanation. These findings are corroborated by studies in the literature (See for example [8]).

PTs who were trying to get the result by making probability calculations, could distinguish, after using simulation, between the theoretical probability gained from probability calculations and the probability prediction gained through simulation. Trials with the simulations helped PTs encounter their opinions before the application and change their ideas, and get convinced about this case. It is reckoned that PTs' active role, utilization of to-the-target materials, intuitions and also trying different ways of making judgements have a great influence. In fact, it is recognised also in the literature that employing teacher-centred approaches, absence of proper teaching materials, and making decisions based on intuitions are major difficulties in probability teaching [5,6,7,29]. In this study, the learning environment was enriched by simulations. Through the observations, it was found that learners' interest increased and they enjoyed learning. Indeed, similar studies enriched with various materials substantiate these findings [29,14].

Use of simulations, by visualising the changes in the experiment outcomes, contributed to the improvement of PTs' ideas about probability, randomness, and variance. Trials with simulations, as well as paving the way to make consistent predictions about the solution of the problem, enabled PTs to think about how to appropriate information from sources, why a model is suitable, and theoretical reasons that define the results. As a matter of fact, it was highlighted that computer-based simulations would provide a strong mathematical basis for teachers in the future, and modern mathematics teaching with opportunities to analyse and represent real cases, meet all important needs such as problem solving and decision-making based on mathematical judgement (NCTM, 2000). Simulations also contributed to create mediums for group or class discussions. With dynamic statistics software, students get the chance to make data-based discussions and inferences that are not possible in data analysis activities they do with pen and paper [30].

By employing many different teaching materials, learners not only organize and structure their knowledge but also experience mathematical concepts and structures in different perspectives [26]. This study concludes that simulations could be very efficient learning and teaching tools to understand abstract concepts related to repetitive random processes, and teach probability in an entertaining way. It is suggested, in this context, that studies should be carried out in order to identify the most effective components of simulations, develop simulations proper to teaching methods in different subjects and find out their influences.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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