The Astrophysical Journal Letters, 890:L16 (3pp), 2020 February 20 https://doi.org/10.3847/2041-8213/ab72ab

Upper Limit on the Dissipation of Gravitational Waves in Gravitationally Bound Systems

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Received 2020 January 6; revised 2020 February 2; accepted 2020 February 4; published 2020 February 14

Abstract

It is shown that a gravitationally bound system with a one-dimensional velocity dispersion σ can at most dissipate a fraction $\sim 36(\sigma/c)^3$ of the gravitational wave (GW) energy propagating through it, even if their dynamical time is shorter than the wave period. The limit is saturated for low-frequency waves propagating through a system of particles with a mean-free-path equal to the size of the system, such as hot protons in galaxy clusters, strongly interacting dark matter particles in halos, or massive black holes in clusters. For such systems with random motions and no resonances, the dissipated fraction, $\lesssim 10^{-6}$, does not degrade the use of GWs as cosmological probes. At high-wave frequencies, the dissipated fraction is additionally suppressed by the square of the ratio between the collision frequency and the wave frequency. The electromagnetic counterparts that result from the dissipation are too faint to be detectable at cosmological distances.

Unified Astronomy Thesaurus concepts: Gravitational waves (678)

1. Introduction

The discovery of gravitational waves (GWs) by LIGO (Abbott et al. 2016) revolutionized observational astronomy by expanding its means for detecting sources at cosmological distances beyond electromagnetic (EM) radiation (Metzger 2019). In particular, GW sources with known redshifts can serve as "standard sirens" (Schutz 1986; Holz & Hughes 2005; Chen et al. 2019) for measuring accurately cosmological distances, while avoiding the uncertainties or systematics of traditional "distance ladder" techniques (Freedman et al. 2019; Riess et al. 2019; Verde et al. 2019; Foley et al. 2020), because the GW source physics is well understood.

An implicit assumption in all past discussions on measuring cosmological distances with GW sources is that the GW signal is not modified as it propagates through intervening matter. This constitutes a key advantage of GWs relative to "standardized candles" of EM radiation, such as SNe Ia (Riess et al. 2019), which could be absorbed by intervening gas and dust along the line of sight (Aguirre 1999).

Nevertheless, a medium with a dynamic viscosity coefficient η could dissipate the energy density of GWs on a dissipation timescale (Hawking 1966; Weinberg 1972),

$$t_{\rm diss} = \frac{c^2}{16\pi G\eta}, \quad (1)$$

where c is the speed of light and G is Newton's constant. To within a factor of order unity, the dynamic viscosity coefficient can be expressed as (Chapman & Cowling 1970)

$$\eta \sim \rho\lambda\sigma, \quad (2)$$

where $\lambda = 1/(nA)$ is the collision mean-free-path, $n = (\rho/m)$ is the particle number density corresponding to a mass density ρ, A is the collision cross-section, $\sigma \equiv \frac{1}{3}\langle v^2\rangle^{1/2}$ is the one-dimensional velocity dispersion, and m is the mass of the particles that make up the dissipative medium. Equations (1)–(2) hold as long as the GW period is larger than the system's dynamical time, so that the particles behave as a fluid during the passage of the GW.

Other effects, such as resonances (Servin et al. 2001; McKernan et al. 2014; Annulli et al. 2018; Montani & Moretti 2019), could enhance the dissipation even in collisionless systems. In particular, the cosmic neutrino background dissipated the energy of primordial GWs by up to 35.6% for comoving wavelengths that entered the horizon during the radiation dominated epoch (Weinberg 2004). One may wonder whether GW dissipation would also be significant in the dense environments of galactic nuclei, where some GW sources are preferentially formed (Loeb 2010; Bartos et al. 2017; Tagawa et al. 2019), even if environmental heating of stars or accretion disks by GW sources is not sufficiently strong to be detectable at extragalactic distances (Kocsis & Loeb 2008; Li et al. 2012). For simplicity, we focus on systems with random motions and no resonances.

As long as the GW period is longer than the system's dynamical time, the dissipation time is minimized for a system with a radius, R, that is comparable to the collision mean-free-path of its particles, λ. Shorter mean-free-paths result in a smaller viscosity coefficient and longer values are not allowed as the particles are confined to the system. Collision rates below the optimal value only reduce the level of dissipation during the passage of the GW through the system.

Examples for optimal systems with $\lambda \sim R$ include hot protons in clusters of galaxies (Loeb 2007), strongly interacting dark matter in halos (Goswami et al. 2017; Fitts et al. 2019), and massive black holes that scatter off each other gravitationally in clusters.[1] The maximal dissipation in these examples would be achieved for primordial GWs of very low frequencies, $\lesssim(\sigma/R) = 1/t_{\rm dyn}$, or for GWs produced by

[1] Interestingly, black holes with masses of order $M_{\rm BH} \sim 10^5 M_\odot$ possess a cross-section per unit mass for gravitational scattering off each other, $(A/m) \sim \pi(GM_{\rm BH}/\sigma^2)^2/M_{\rm BH}$, which overlaps with the value of $(A/m) \sim 1\ {\rm cm^2\ g^{-1}}$ needed to alleviate the cusp-core problem in dwarf galaxies, as it provides $\lambda \sim R$ at relative speeds of $\sigma \sim 10\ {\rm km\ s^{-1}}$. The velocity scaling, $(A/m) \propto \sigma^{-4}$, reduces the collisional effect in more massive halos, as envisioned for dark matter with a Yukawa potential (Loeb & Weiner 2011). Unfortunately, massive black holes cannot serve as primary candidates for strongly interacting dark matter based on other constraints (Carr 2019). But a cluster of them can dissipate GW energy by converting it into an increase in σ ("heat") through two-body scatterings.

binaries with an orbital period longer than the dynamical time of the absorbing system, $t_{\rm dyn}$.

This brief note sets an upper limit on the level of dissipation that a GW signal encounters by passing through astrophysical systems that are bound by gravity for arbitrary GW frequencies. The limit is independent of the composition or nature of the absorbing medium as long as there are no resonances with the GW frequency (Servin et al. 2001; McKernan et al. 2014; Annulli et al. 2018; Montani & Moretti 2019). Its general validity clears the way for using GW sources for precise cosmological measurements by observatories such as LIGO/Virgo,[2] LISA,[3] or their future extensions (Hall & Evans 2019).

The frequency-independent expressions (1)–(2) are valid as long as the GW frequency is smaller than the collision frequency of particles in the system. Otherwise, dissipation is suppressed because particles have a low collision probability per GW period, after which they return to their original position and velocity with no memory of previous oscillations. In this high GW frequency regime, the velocity shear being dissipated is dictated by the amplitude of the periodic motion of the particles. We derive this additional (frequency-dependent) suppression of the GW dissipation in the concluding section.

2. Absolute Dissipation Limit for Arbitrary GW Frequency

Let us consider a collisional system of radius R, composed of particles that are bound by gravity, without making any assumptions about the nature of the constituent particles. A GW signal would cross the system over a timescale $t_{\rm cross} \sim (R/c)$, which is shorter than the crossing-time by the system particles, $\sim(R/\sigma)$. During the GW passage, viscous dissipation is maximized for $\lambda \sim R$, as already noted. Larger values of the mean-free-path, λ, are not allowed because particle trajectories are gravitationally confined to the system size. Smaller values of λ reduce the viscosity coefficient based on Equation (2).

The fraction of the GW energy which the system absorbs is

$$\epsilon_{\rm diss} \sim \frac{t_{\rm cross}}{t_{\rm diss}}. \quad (3)$$

Substituting the maximum viscosity coefficient, $\eta \sim \rho R\sigma$, into Equation (1), yields an upper limit on the dissipated fraction of the GW energy,

$$\epsilon_{\rm diss} < (16\pi)\frac{G\rho R^2\sigma}{c^3}. \quad (4)$$

For a system bound by the gravitational potential of the dissipating particles plus other components, such as gas, stars, black holes, or dark matter, the Virial Theorem implies (Binney & Tremaine 2008)

$$\left(\frac{4\pi}{3}\right)G\rho R^2 < 3\sigma^2, \quad (5)$$

where the inequality stems from the fact that the dissipating particles with a mean mass density $\rho = M(<R)/[(4\pi/3)R^3]$ account for only a fraction of the total mass density in the system, which could include additional components. Substituting (5) into (4) yields our final upper limit:

$$\epsilon_{\rm diss} < 36\left(\frac{\sigma}{c}\right)^3. \quad (6)$$

The numerical coefficient on the right-hand side of (6) could change by a factor of order unity, depending on the detailed radial profile of ρ and λ within the system.

3. EM Counterparts

The final result (6) implies that the fraction of GW energy that can be absorbed by any self-gravitating system of a one-dimensional velocity dispersion σ is limited to $\sim 36(\sigma/c)^3$. Dark matter halos possess a maximum value of $(\sigma/c) \lesssim 10^{-2.5}$ in clusters of galaxies (Loeb & Furlanetto 2013), and cannot dissipate more than $\sim 10^{-6}$ of the the GW energy from a source hosted by them or located behind them. This limit applies to all possible values of the self-interaction cross-section per unit mass of dark matter particles at all GW frequencies.

The negative heat capacity of gravitationally bound systems makes them vulnerable to the gravothermal instability (Balberg & Shapiro 2002; Hennawi & Ostriker 2002). As a result, compact systems with large values of (σ/c) and a short collisional mean-free-path, $\lambda \ll R$, could evolve to a black hole or evaporate on a timescale shorter than the age of the universe.

Consequently, the amplitude of GW signals cannot be absorbed by intervening gravitationally bound systems to any significant level that would degrade their potential use for cosmology (Schutz 1986; Holz & Hughes 2005; Chen et al. 2019). In particular, uncertainties in the peculiar velocities of GW sources are of order $\sim\sigma/c$ and exceed by a factor $\gtrsim 0.03(c/\sigma)^2$ the level of viscous dissipation within their host dynamical system.

The above results also limit a possible EM counterpart to the GW signal from its environment (Kocsis & Loeb 2008; Li et al. 2012), unrelated to the possible EM emission by the source itself (Loeb 2016; D'Orazio & Loeb 2018; Metzger 2019). The dissipation of a fraction $\epsilon_{\rm diss}$ of the GW energy, $E_{\rm GW}$, in a baryonic system surrounding the GW source, would lead to an EM counterpart with a luminosity

$$L_{\rm EM} \sim \epsilon_{\rm cool}\,\epsilon_{\rm diss}\left(\frac{E_{\rm GW}}{t_{\rm cool}}\right), \quad (7)$$

where $\epsilon_{\rm cool}$ is the fraction of the dissipated energy that gets radiated electromagnetically over a cooling time, $t_{\rm cool}$. The time delay across the system sets a lower limit on the cooling time, $t_{\rm cool} \gtrsim (R/c)$, and hence an upper limit on the EM luminosity based on (6) and (7) for the ultimate radiative efficiency of $\epsilon_{\rm cool} \sim 1$,

$$L_{\rm EM,max} \lesssim 10^{40}\ \frac{\rm erg}{\rm s}\left(\frac{E_{\rm GW}}{0.1 M_\odot\, c^2}\right)\left(\frac{\sigma}{10^{-3}\, c}\right)^3\left(\frac{R}{0.01\ \rm pc}\right)^{-1}, \quad (8)$$

over a period of $\gtrsim$10 days $(R/0.01\ {\rm pc})$. The limit is nearly 20 orders of magnitude below the maximum attainable GW luminosity, $\sim(c^5/G) = 4 \times 10^{59}\ {\rm erg\ s^{-1}}$. It can also be normalized by the Eddington EM limit for the total mass $M_{\rm tot}$ of the host dynamical system, $L_{\rm Edd} = 1.4 \times 10^{44}\ {\rm erg\ s^{-1}}$ $(M_{\rm tot}/10^6\, M_\odot)$ (Loeb & Furlanetto 2013). Using the *Virial Theorem* again, $(GM_{\rm tot}/R) \sim 3\sigma^2$, the normalized upper limit is tight,

[2] https://www.ligo.org/
[3] https://www.elisascience.org/

$$\frac{L_{\rm EM,max}}{L_{\rm Edd}} \lesssim 10^{-4}\left(\frac{E_{\rm GW}}{0.1\,M_\odot\,c^2}\right)\left(\frac{\sigma}{10^{-3}\,c}\right)\left(\frac{R}{0.01\ {\rm pc}}\right)^{-2}, \quad (9)$$

implying that EM counterparts from viscous dissipation of GW signals at cosmological distances are too faint to be detectable by existing telescopes.

4. Further Suppression at High GW Frequencies

The above limits were derived without a reference to the GW frequency. However, the frequency-independent dissipation rate in Equation (1) could be amplified by resonances of the GW frequency with modes in the medium, such as those associated with binary systems (McKernan et al. 2014; Annulli et al. 2018; Montani & Moretti 2019) or a magnetic field (Servin et al. 2001).

In thermal systems with random motions of particles, the standard viscous dissipation rate increases with increasing mean-free-time between collisions because particles are able to sample a steadily increasing velocity offset in the underlying shear flow. As the GW-induced shear reverses sign on the GW period, the fact that a particle waits longer than a wave period for the next collision does not help it develop more velocity offset relative to the local flow. The maximum shear that it samples is the value that the GW induces over a single wave period, and this fixed amount is dissipated over the collision period. This is in contrast to the behavior at short collision periods where the shear sampled is inversely proportional to the collision period, yielding a dissipation rate in (1) that is proportional to the viscosity coefficient.[4]

Equation (6) provides the absolute upper limit on $\epsilon_{\rm diss}$ for any GW frequency by considering the maximum possible value of η, but the actual limit on $\epsilon_{\rm diss}$ at high GW frequencies is tighter by the square of the ratio between the lower collision frequency and the GW frequency. This can be derived as follows.

In analogy with the propagation of EM waves in a collisional plasma (Braginskii 1965; Stix 1992), the introduction of a *Crook collision term*, $-\nu_{\rm coll}\boldsymbol{v}$, to the momentum equation describing the acceleration of a particle, $d\boldsymbol{v}/dt$, by a GW that oscillates over time t as $\propto e^{i\omega t}$ with a GW frequency ω, leads to a dissipation rate that rises inversely with the frequency ratio,

$$f_{\rm ratio} = \left(\frac{\nu_{\rm coll}}{\omega}\right), \quad (10)$$

for $f_{\rm ratio} \ll 1$ (consistently with Equation (1)), peaks at $f_{\rm ratio} \sim 1$ and then declines in proportion to $f_{\rm ratio}^{-1}$ for $f_{\rm ratio} \gg 1$.

At high GW frequencies, the above formulation provides an additional (frequency-dependent) suppression factor of,

$$\frac{1}{1 + f_{\rm ratio}^{-2}}, \quad (11)$$

on the right-hand-side of the limit (6). This suppression factor does not depend on the nature of the periodic driving force (be it EM or GW) but only on the collisional dynamics of the particles in the system. This extra suppression factor obtains extremely small values at the GW frequencies detectable by LIGO and most dissipating systems.

In the transition regime, where $f_{\rm ratio} \sim 1$, the dissipated fraction is limited by

$$\epsilon_{\rm diss} < 36\left(\frac{\sigma}{c}\right)^3\left(\frac{1}{\omega t_{\rm dyn}}\right). \quad (12)$$

where we have used Equations (1)–(3) and (5), and the relations $t_{\rm dyn} = (R/\sigma)$ and $\nu_{\rm coll} = (\sigma/\lambda)$. This limit applies only for $\omega t_{\rm dyn} \gtrsim 1$, with the limit (6) being saturated at lower frequencies.

My special thanks go to Bence Kocsis and Martin Rees for inspiring correspondences that led to the addition of the last section on high GW frequencies. After the completion of this addition, I received a research note from Jeremy Goodman through the AAS Editorial office, in which the excess suppression at high GW frequencies was derived independently based on an elegant mathematical formalism. I am deeply grateful to Jeremy for his interest in my manuscript and for communicating his insightful derivation. This work was supported in part by the *black hole Initiative* at Harvard University, which is funded by grants from GBMF and JTF.

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[4] Note that the collision period must be compared relative to the GW period rather than the mean-free-path relative to the GW wavelength. The two criteria are different for non-relativistic particles.