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Pair Trading: Random Weight Approach

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

Pairs trading are standard approaches for statistical arbitrage detection. The logic behind pair trading approach is to construct a portfolio of two financial assets with special weights where this portfolio has zero value in time zero and creates positive value with a high probability in a certain future. In this paper, a random weight approach is introduced for construction of pairs trading strategies. Random weight approach is a method at which the weights of each asset comes from a statistical distribution. In this note, the beta distribution is selected. Under this circumstance, first, condition for market neutrality of portfolio is considered then, the probability of attaining the profit is maximized. The portfolio contains long position in random units of first asset and random units of another asset in short position. Strategies are given and parameters selections for maximizing the probability of profit are proposed.

Keywords: Beta distribution; pairs trading; random weight; single factor models; statistical arbitrage.

1 Introduction

The pair trading is one of market neutral strategies which belong to statistical arbitrage strategies. The certain arbitrage opportunity searches for a certain risk-free profit in future with zero investing at the current time. However, statistical arbitrage searches for arbitrage opportunities which exist with a high probability. Indeed, statistical arbitrage strategies conjectures statistical mispricings of price relationships that are true in expectation, in the long run when repeating a trading strategy. There is some standard approach like optimal

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control to detect the arbitrage opportunities, see Intriligator [1]. Neutral portfolios with respect to market contains two securities with a long position in one security and a short position in the other, in a suitable ratio [2]. Jarrow et al. [3] studied the relationship between market efficiency and arbitrage opportunities. A comprehensive reference in statistical arbitrage detection in financial markets is Pole [4]. Avellaneda and Lee [5] considered the statistical arbitrage opportunities in US market of equities. Goldberg [6] surveyed the shortest path algorithms to detect arbitrage opportunities in foreign exchange markets. Aldridge [7] studied the application of high frequency approach as a practical guide to algorithmic trading systems.

An important problem in pair trading is the identification of pair and an efficient trading algorithm. In this note, the random weight approach is discussed to obtain the pairs trading strategy. Random weight approach is a method at which the weights of assets come from a statistical distribution. In this note, the beta distribution is selected. One reason for selecting the beta distribution is its shape flexibility. By changing the parameters of beta distribution, most of distributions on [0,1] may be approximated (see, Zhang and Wu, [8]). To this end, consider two capital assets, where at time t, their return satisfy in single factor models as follows,

$$\begin{cases} R_{1t} = \alpha_{1t} + \beta_{1t} R_{mt} + \varepsilon_{1t,} \\ R_{2t} = \alpha_{2t} + \beta_{2t} R_{mt} + \varepsilon_{2t}. \end{cases} \tag{1}$$

In equation (1), R_{mt} is the return of market. Here, β's mean the market risk indicator (systematic risk index) and α's are the intercept of models. The single factor model is a simple regression model between return of asset and return of market. In practice, parameters of above model are estimated using suitable estimation methods like the least square or the maximum likelihood methods, hence, in this paper, it is assumed that parameters $\alpha_{it}, \beta_{it}, i = 1,2$ are known. Variables ε_{it} have known distribution F_{it} with mean μ_{it} and variance $\sigma_{it}^2, i = 1,2$. They are mutually independent. Let γ_t be the weight of first asset and assume that it is a beta distributed random variable Mood et al. [9] with parameters $\theta_t$and τ_t independent of all variables exist in the both models. Next, consider a portfolio of long position in γ_t units of R_{1t} and short position in $(1-\gamma_t)$ units of R_{2t}. Return of related portfolio is

$$R_{pt} = \gamma_t R_{1t} - (1-\gamma_t) R_{2t} \tag{2}$$

At time zero, γ_0 is chosen such that $R_{p0} = 0$ that is $\gamma_0 = \frac{R_{20}}{R_{10}+R_{20}}$. At time t, γ_t, R_{1t} and R_{2t} are chosen such that $P(R_{pt} > 0)$ is close to one. The following propositions studies the conditions of market neutrality and a Monte Carlo estimate for the probability of statistical arbitrage $\pi_t = P\big(R_{pt} > 0\big)$.

Proposition 1: The expected return of portfolio $E(R_{pt})$ is independent of $E(R_{mt})$ if and only if $\tau_t = \frac{\beta_{1t}}{\beta_{2t}} \theta_t$, in this case,

$$E(R_{pt}) = \frac{\alpha_{1t}\theta_t - \alpha_{2t}\tau_t}{\theta_t + \tau_t}. \tag{3}$$

Proof: The coefficient of $E(R_{mt})$ (equation 3) in $E(R_{pt})$ is $\frac{\theta_t\beta_{1t} - \beta_{2t}\tau_t}{\theta_t + \tau_t}$. This is zero if and only if $\theta_t\beta_{1t} - \beta_{2t}\tau_t = 0$. Therefore, $E(R_{pt}) = \frac{\alpha_{1t}\theta_t - \alpha_{2t}\tau_t}{\theta_t + \tau_t}$. This completes the proof.

Proposition 2: Let $\pi_t = P\big(R_{pt} > 0\big) = P\big(\gamma_t > Z_t\big) = 1 - E\left(F_{\gamma_t}(Z_t)\right)$ where $Z_t = \frac{R_{2t}}{R_{1t}+R_{2t}}$. The Monte Carlo estimate of π_t is

$$\hat{\pi}_t = 1 - \frac{1}{R}\sum_{i=1}^{R} F_{\gamma_t}(Z_{it}), \tag{4}$$

where $Z_{it}, i = 1,2, \dots, R$ is an iid sample from Z_t in equation 4.

Proof: The proof is straightforward by applying the standard Monte Carlo method Mood et al. [9] for $E\left(F_{\gamma_t}(Z_t)\right)$.

Remark 1: To make sure that $\hat{\pi}_t \to 1$, almost sure, it is enough to determine $\theta_t$and τ_t such that

$$\frac{1}{R}\sum_{i=1}^{R} F_{\gamma_t}(Z_{it}) \to 0, \text{ a.s.} \tag{5}$$

A pair selection strategy is to select R_{1t} and R_{2t} such that $R_{1t} \to 0$ and $R_{1t} \to \infty$, a.s. Thus, when the equation 5 is true, then $\hat{\pi}_t \to 1$.

The rest of paper is organized as follows. The portfolio construction strategies are given in the next section. Section 3 concludes.

2 Portfolio Constructions

In previous section, it was seen that the portfolio constructed as (equation 2), $R_{pt} = \gamma_t R_{1t} - (1-\gamma_t)R_{2t}$ has a high probability for profitability, for each fixed t. An important question is which values of (0,1) should be chosen for γ. The solution is to find θ and τ such that $P(\gamma > z) = 1 - \alpha$ for some small α. Thus, z is the α −the quantile of distribution of γ. Thus, generate $z_1, \dots, z_R$ and calculate their average $\bar{z}$. Find θ and τ such that $\bar{z}$ is the α −the quantile of distribution of γ. Then, generate one sample from this distribution, a suitable value for γ is this number.

Example 1: In this example, assume that $\alpha_1 = 0.5, \alpha_2 = 0.75, \beta_1 = 0.5, \beta_2 = 1.7$. Variables ε_1 and ε_2 have normal distributions with zero means and variances 0.01 and 0.36. The market return has normal distribution with mean 1.5 and variance 0.16. $\theta \in \{0,0.1,0.02,\dots,5\}$ and $\tau = \frac{\beta_1}{\beta_2}\theta$. Here, the time series plot of π_k is drawn (Fig. 1). The number of repetition is $R = 1000$. This shows the high probability of existing arbitrage opportunity. The horizontal axis is time and vertical axis is the π_k.

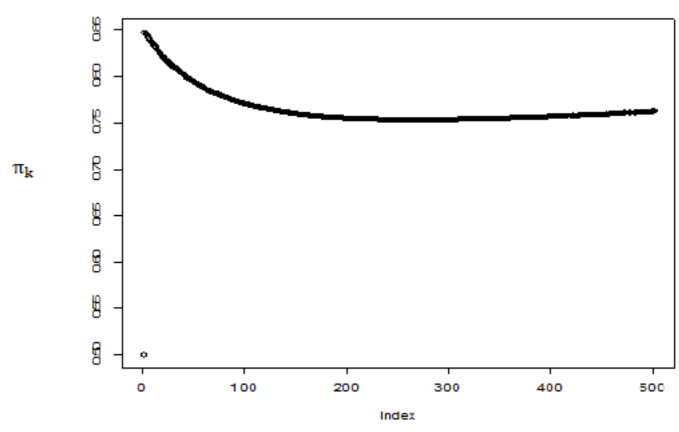

Fig. 1. Probability of statistical arbitrage

Example 2: Here, for various α_i and β_i's suitable values for θ and τ are computed in Table 1. Again, it is assumed that ε_1 and ε_2 have normal distributions with zero means and variances 0.01 and 0.36. The market return has normal distribution with mean 1.5 and variance 0.16. here, it is assumed that $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. Parameters θ and τ are computed such that the probability of statistical arbitrage is maximized.

Table 1. Values for θ and τ

α_1	β_1	θ	τ
0.5	0.6	3.98	3.98
0.7	0.8	5	5
0.9	0.7	5	5
1.1	0.9	5	5
1.1	1.3	5	5
1.5	0.04	5	5
0.06	0.03	0	0

Conditions are derived to ensure the probability of statistical arbitrage to high. Time t is fixed and therefore it is dropped from notation. Define $k_1 = \frac{\alpha_1+\alpha_2}{\alpha_2}$, $k_2 = \frac{\beta_1+\beta_2}{\beta_2}$, $k_3 = \frac{\sigma_1}{\sigma_2}$. Suppose that $\alpha_2 \to 0$ and $k_1 \to 1$. Then, parameter $\frac{\alpha_1}{\alpha_2}$ converges to zero, i.e., $\frac{\alpha_1}{\alpha_2} \to 0$. Similar conditions is held for k_2 and β_1, β_2. Also, suppose that k_3 does not converge to zero and bounded away zero. It is easy to see that

$$z \approx \frac{\varepsilon_2^*}{\varepsilon_2^*+k_3\varepsilon_1^*}, \tag{6}$$

almost sure, where ε_1^* and ε_2^* have standard normal distributions, in equation 6. If k_3 is too far from zero, then z has Cauchy distribution with infinite mean and variance. Thus, in this case, z gets small and large values infinitely often. So, it is reasonable to suggest the investor to trade when z is small. To see the Cauchy distribution of z Mood et al. [9], notice that for arbitrary number c, we have

$$z - c = \frac{\varepsilon_2^*}{\varepsilon_2^*+k_3\varepsilon_1^*} - c = \frac{(1-c)\varepsilon_2^* - k_3 c\varepsilon_1^*}{\varepsilon_2^*+k_3\varepsilon_1^*}. \tag{7}$$

Proposition 3: Let $k_1 = \frac{\alpha_1+\alpha_2}{\alpha_2}$, $k_2 = \frac{\beta_1+\beta_2}{\beta_2}$, $k_3 = \frac{\sigma_1}{\sigma_2}$. Suppose that $\alpha_2 \to 0$ and $k_1 \to 1$. That is, $\frac{\alpha_1}{\alpha_2} \to 0$. The similar conditions happens for k_2 and β_1, β_2. Also, suppose that k_3 does not converge to zero and bounded away zero. Generate a random sample from z and calculate U. Hence, trade when U is small.

Proof: Determine c such that two linear combination of two independent normal variables be orthogonal. That is $(1-c) - k_3^2 c = 0$. Then, $c = \frac{1}{1+k_3^2}$. Then,

$$U = \sqrt{\frac{1+k_3^2}{(1-c)^2 + k_3^2 c^2}}\, z = \frac{1+k_3^2}{k_3^2} z$$

is the fraction of two independent standard normal variables, therefore it is a Cauchy distributed random variable. Notice that

$$\pi_t = P(\gamma_t > z) = P\left(\gamma_t > \frac{k_3^2}{1+k_3^2} U\right). \tag{8}$$

Next, calculate U, when it is small the π_t is high. The above formulae (equation 8) gives an useful expression for the probability of statistical arbitrage.

3 Conclusions

Combination of random weighting and pairs trading approaches is applied for detection of statistical arbitrage. The market contains two asset one in long and another in short position with random weights. The market neutrality conditions are derived such that the probability of attaining the profit is maximized.

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Competing Interests

Author has declared that no competing interests exist.

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