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Energy of Fuzzy Regular and Graceful Graphs

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Authors' contributions

This work carried out in collaboration between both authors. Author AN designed the study, derive the adjacency matrix, energy and wrote the first draft of manuscript. Author SV analyzed the manuscript, corrected the manuscript and chose the best journal for publication. Both authors read and approved the final manuscript.

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Abstract

Energy of graph and energy of fuzzy graph is the sum of the absolute values of the eigen values of adjacency matrix. The concept of energy of fuzzy graph is extended to fuzzy regular, totally regular and graceful graphs in this paper. This paper intends to elaborate about characteristics of eigen values, upper and lower bound of energy.

Keywords: Eigen values; energy; regular; totally regular; graceful graphs.

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1 Introduction

Energy of graphs was first defined by Ivan Gutman in 1978.Organic molecules can be represented by graphs called molecular graphs. Energy of different graphs including regular [1], non-regular [2], circulate [3] and

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random graphs [4] is also under study. Energy is defined for signed graphs in [5] and for weighted graph by I. Gutman and Shao in 2011. In [6] R. Balakrishnan defined the energy of π electrons of the molecule is approximately the energy of its molecular graph.

Fuzzy graphs are encountered in fuzzy set theory. A fuzzy set was defined by Zadeh in 1965 [7]. Every element in the universal set is assigned a grade of membership in [0,1]. Fuzzy set are representation of how a human brain perceives the objects in the world. Hence, fuzzy set theory and fuzzy graph theory have many applications in those areas are in [8] solving minimum spanning tree problem in fuzzy environment, In [9] Fuzzy Set in Fuzzy Shortest Path Problem. 1975 Rosen field developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts like fuzzy line graph, fuzzy cycle [10], established some characterization of fuzzy tree using fuzzy bridges and fuzzy cut nodes and also introduced the concept of bipartite fuzzy graph [11]. In [12] Anjali Narayanan and Sunil Mathew introduced concept of the energy of fuzzy graphs. In [13,14] S. Vimala introduced the concept of energy of fuzzy labeling graph. In [15] A. Pal, and T. Pal, discusses about the fuzzy robust graph coloring problem. In [16] A. Dey and A. Pal, introduced Vertex coloring of a fuzzy graph using alpha cut.

In this paper, a comparative study made between regular and totally regular graph, and graceful graphs though various examples. Then a characterization of eigen values provided. Also, bound on the energy is studied.

2 Preliminaries

2.1 Fuzzy graph: [13]

Let U and V be two sets. Then ρ is said to be a fuzzy relations from $U \times V$ to $[0,1]$. A fuzzy graph $G = (\sigma, \mu)$ is a pair of function of vertices and edges consider $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

2.2 Bounds of energy

$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$.

Lower bound > Upper bound > Energy

2.3 Regular graph: [13]

Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_G(V) = k$, for all $v \in V$. (i.e.), if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph.

2.4 Totally regular graph: [13]

Let $G(\sigma, \mu)$ be a fuzzy graph on G^*. The total degree of a vertex $u \in V$ is defined by

$$Ed_G(V) = \sum_{u \in v} \mu(uv) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u) = d_G(V) + \sigma(u) \qquad (2)$$

If each vertex of g has the same total degree k, then G is said to be a totally regular fuzzy graphs.

2.5 Labeling: [2]

A labeling of a graph is an assignment of values to the vertices and edges of a graph. Given a graph G, an injective function $f: V(G) \rightarrow N$ has been called a vertex labeling of G. An edge labeling of a graph is a bijection from $E(G)$ to the set $\{1, 2, ..., |E(G)|\}$.

2.6 Fuzzy labeling graph: [14,15]

A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph, if $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

2.7 Graceful labeling: [2]

A graph G with q edges if f is an injection from the vertices of G to the set 0,1,…, q such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct such labeling called graceful.

2.8 Fuzzy graceful labeling

A graceful labeling admits fuzzy values it’s called fuzzy graceful labeling.

3 Main Results

Theorem: 3.1

If a fuzzy graph G is both regular and totally regular, then the eigen values are balanced on the energy (i.e.).,

$$\sum_{i=1}^{n} \pm\lambda_i = 0 \qquad (3)$$

Proof:

Let G be a k_1-regular and k_2-totally regular fuzzy graph.

Hence $d(u) = k_1$ for all $u \in V$ and $td(u) = k_2$ for all $u \in V$.

Let $v_1, v_2, ..., v_n$ be a vertices of G. It can be represented by an $n \times n$ matrix giving the adjacency between the vertices.

Calculate eigen values $\pm\lambda_1, \pm\lambda_2, ..., \pm\lambda_n$

$$\sum_{i=1}^{n} \pm\lambda_i = 0$$

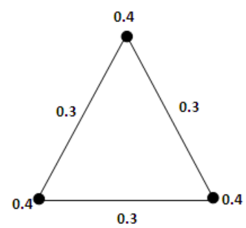
Therefore the eigen values are balance on the energy in regular and totally regular fuzzy graphs.

Remark: 3.1.1

A regular and totally regular fuzzy graphs are satisfying then the eigen values are balanced for n≥3.

Example: 3.1.2

Consider $G^*:(V,E)$ where $V=\{v_1,v_2,v_3\}$ and $E=\{v_1,v_2,v_1v_3\}$. Define $G:(\sigma,\mu)$ by $\sigma(v_1)=0.3$, $\sigma(v_2)=\sigma(v_3)=0.4$ and $\mu(v_1v_2)=0.3$, $\mu(v_1v_3)=0.3$, $\mu(v_2v_3)=0.3$



$$A(G)=\begin{bmatrix} 0 & 0.3 & 0.3 \\ 0.3 & 0 & 0.3 \\ 0.3 & 0.3 & 0 \end{bmatrix}$$

Then $d(v_i)=0.6$ for all $i=1,2,3$. So, G is regular fuzzy graph. Also $d(v_i)=1$ for all $i=1,2,3$. Hence G is also a totally fuzzy graph.

Eigen values are: -0.3000,-0.3000, 0.6000 and -0.3000 - 0.3000 + 0.6000 = 0,

$$\sum_{i=1}^{n}\pm\lambda_i=0.$$

Energy of graph $\sum_{i=1}^{n}|\lambda_i|=$ (0.3000 + 0.3000 + 0.6000) = 1.20

. $\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}}$ > $\sqrt{2(\sum_{i=1}^{m} m_i^2)n}$ > $EF_l(G)$.

In this case 1.062 >1.09 > 1.2 .

Lower bound >Upper bound >Energy

Theorem: 3.2

Let G be a fuzzy graceful graphs then the eigen values are balanced $\sum_{i=1}^{n}\pm\lambda_i=0$.

Proof:

Let G be a fuzzy graceful graph when each edge xy label by $|f(x)-f(y)|$ the resulting edge label are distinct.
Let $v_1, v_2, ..., v_n$ be a vertices of G. It can be represented by an $n \times n$ matrix giving the adjacency between the vertices.

Calculate eigen values $\pm\lambda_1, \pm\lambda_2, ..., \pm\lambda_n$

$$\sum_{i=1}^{n} \pm\lambda_i = 0$$

Therefore the eigen values are balance on the energy in fuzzy graceful graphs.

Remark: 3.2.1

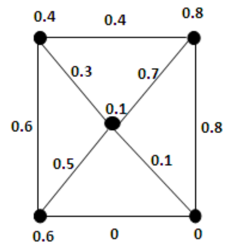
A fuzzy graceful wheel graph and fuzzy graceful complete graph are satisfying that the eigen value are balanced for $n \geq 1, 3$ in complete graph and $n \geq 5$ in wheel graphs.

Example: 3.2.2

Consider $G^* : (V, E)$ be a fuzzy graceful wheel graph where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and

$$E = \{v_1v_2, v_1v_5, v_1v_3, v_1v_4, v_2v_5, v_2v_3, v_3v_4, v_4v_5\}.$$

Define $G : (\sigma, \mu)$ by $\sigma(v_1) = 0.1,\ \sigma(v_2) = 0.4,\ \sigma(v_3) = 0.8,\ \sigma(v_4) = 0,\ \sigma(v_5) = 0.6$ and $\mu(v_1v_2) = 0.3,\ \mu(v_1v_3) = 0.7,\ \mu(v_1v_4) = 0.1,\ \mu(v_2v_3) = 0.4,\ \mu(v_3v_4) = 0.8,\ \mu(v_4v_5) = 0,$ $\mu(v_1v_5) = 0.5,\ \mu(v_5v_1) = 0.6$.



$$A(G) = \begin{bmatrix} 0 & 0.7 & 0.1 & 0.5 & 0.3 \\ 0.7 & 0 & 0.8 & 0 & 0.4 \\ 0.1 & 0.8 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0.6 \\ 0.3 & 0.4 & 0 & 0.6 & 0 \end{bmatrix}$$

Eigen values are: -1.2992, -0.3195, -0.0228, 0.1344, 1.5071.

-1.2992 - 0.3195 - 0.0228 + 0.1344 + 1.5071 = 0,

(i.e): $\sum_{i=1}^{n} \pm\lambda_i = 0$

Energy of graph $\sum_{i=1}^{n} |\lambda_i| =$ (1.2992 + 0.3195 + 0.0228 + 0.1344 + 1.5071) = 3.283

$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$.

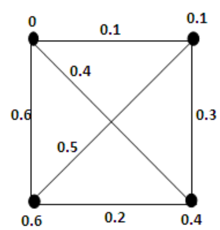
1.9160<3.1937< 3.283

Upper bound < Lower bound < Energy

Example: 3.2.3

Consider $G^*:(V,E)$ be a fuzzy graceful complete graph where $V=\{v_1,v_2,v_3,v_4\}$ and $E=\{v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_1v_3, v_2v_4\}$.

Define $G:(\sigma,\mu)$ by $\sigma(v_1)=0,\ \sigma(v_2)=0.1,\ \sigma(v_3)=0.4,\ \sigma(v_4)=0.6$ and $\mu(v_1v_2)=0.1,\ \mu(v_2v_3)=0.3,\ \mu(v_3v_4)=0.2,\ \mu(v_4v_1)=0.6,\ \mu(v_1v_3)=0.4,\ \mu(v_2v_4)=0$.



$$A(G)=\begin{bmatrix} 0 & 0.1 & 0.4 & 0.6 \\ 0.1 & 0 & 0.3 & 0.5 \\ 0.4 & 0.3 & 0 & 0.2 \\ 0.6 & 0.5 & 0.2 & 0 \end{bmatrix}$$

Eigen values are =-0.7941,-0.1854,-0.0914, 1.0709.

-0.7941 - 0.1854 - 0.0914 + 1.0709 = 0

(i.e) $\sum_{i=1}^{n} \pm\lambda_i = 0$

Energy of graph $\sum_{i=1}^{n} |\lambda_i| =$(0.7941 + 0.1854 + 0.0914 + 1.0709) = 2.1417

$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G).$
1.2649<1.3564<2.1417

Upper bound<Lower bound<Energy

Table 1. List of energy and bounds

Name of the graphs	$E(G)$	$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,\lvert A\rvert^{\frac{2}{n}}}$	$\sqrt{2(\sum_{i=1}^{m} m_i^2)n}$
Graceful complete graph K_1	0.2000	0.1732	0.1414
Graceful Complete graph K_3	0.8226	0.6718	0.6486
Graceful Complete graph K_4	2.1417	1.9078	1.5329
Graceful wheel graph W_5	3.5832	3.1937	1.9160
Graceful wheel graph W_6	4.2830	4.1424	4.053
Graceful wheel graph W_7	4.9972	4.4172	3.4370

4 Conclusion

In this paper, we find some energy of fuzzy regular and fuzzy graceful graphs. Our future work is to apply energy in many fuzzy graphs.

Competing Interests

Authors have declared that no competing interests exist.

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