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Review

Bubble dynamics and interface phenomenon

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The dispersion of gas into liquid is a complex part and it is depending on bubble size and its distribution including coalescence and breakup. The study of flow pattern, bubble size and its distribution and factors affecting the bubble size was reviewed by several authors. Various measurement techniques of bubble size are also reported in literature. In this paper we have made an attempt to show the literature survey on bubble dynamics as it plays very crucial role in mass transfer characteristics.

Key words: Computational fluid dynamics (CFD), multiphase flow gas-liquid two-phase flow, bubbly flow, bubble size distribution, Sauter-mean bubble diameter, mean bubble size, bubble breakup and coalescence, free surface flow, slug flow.

INTRODUCTION

Two-phase flow is defined as flow of a heterogeneous mixture of gas and liquid, where the fluid can be identified as macroscopic structure or in other words the fluids in a two-phase flow are not homogeneously mixed at a molecular level, but macroscopic regions of the fluid like droplets, bubbles, slugs, liquid films, ligaments, etc. can be observed. Typical examples of gas-liquid multiphase flow are bubbly, spray, and stratified flow where fluids are separated by a free surface like in annular and slug flow regime of gas-liquid two-phase flow in pipes and channels. Two-phase flow plays an important role in mass and heat transfer. Particularly for mass transfer operation high interfacial area is of most concern. To create high interfacial area the dispersion of one fluid into another is required. To study the dispersion of gas in liquid the knowledge of bubble size and bubble size distribution, bubble breakup and coalescence processes is necessary. Dispersion of one fluid into another is a complex phenomenon and is dependent on many factors like velocity of jet, pressure difference, geometry of nozzle, temperature of both fluids, properties of fluid like their density, viscosity, surface tension, vapor pressure, etc. In this article we have made an attempt to see the article related to bubble dynamics.

FLOW PATTERN

Gas-liquid two-phase flow can be classified in four types: (a) Homogeneous bubbly flow (b) Heterogeneous churn flow (c) Slug flow and (d) Annular flow as shown in Figure 1, and further summarized in Table 1.

Some researchers have further extended the classification to include froth, mist flow, etc. Frank (2005) explained that disperse bubbly flow is characterized by a characteristic bubble diameter (mono dispersed bubbly flow). Disperse bubbly flows have small to moderate gas volume fraction; bubbles have varied shape (spherical, ellipsoidal, spherical cap bubbles). Hence the development of mathematical model must consider the flow morphology of disperse bubbly flow.

Although, flow regime primarily depends upon the gas superficial velocity and column diameter, liquid viscosity sometimes plays the prime role. In bubbly flow, the bubbles are quite uniform in size and they move in an orderly fashion with little collision among bubbles and the liquid is mildly stirred by the bubbles. Yamagiwa et al. (1990) observed that in case of co-current down flow, flow behavior changed from non-uniform bubbling flow to uniform bubbling flow when superficial liquid velocity increases and then to churn turbulent flow. This uniform bubbling flow was again obtained with further increase of liquid velocity.

Kedoush and Al-Khatab (1989) studied flow patterns with air-water flow in 3.8 cm I.D. pipe. They reported that transition from bubbly to slug flow occurs when $\varepsilon_G = 0.3$ (ε_G is gas hold up) and slug flow appears in the range $0.3 \leq \varepsilon_G \leq 0.7$. Mandal et al. (2004) studied

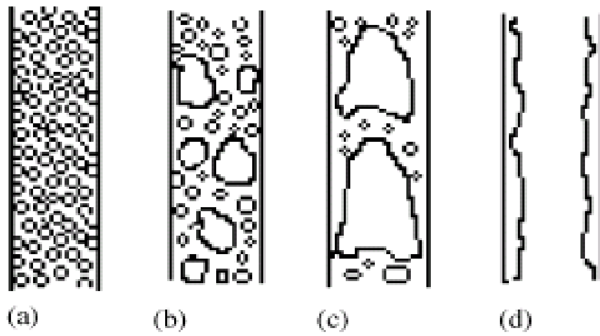

Figure 1. Flow pattern in vertical column [(a) homogeneous bubbly flow (b) heterogeneous churn flow, (c) slug flow and (d) annular flow] (Mandal et al., 2004).

Table 1. Flow patterns for gas-liquid two-phase flow in horizontal and vertical pipes: dependence on the gas volume fraction (Frank, 2005).

Gas volume fraction, (ε_G)	Horizontal pipe flow	Vertical pipe flow
	Finely dispersed bubbly flow	Finely dispersed bubbly flow
Small gas volume fraction		
	Slug flow / plug flow	Disperse bubbly flow with near wall void fraction maximum
↓		Disperse bubbly flow with breakup & coalescence; gas volume fraction core peak
	Stratified flow with free surface (smooth, wavy, etc.)	Taylor bubble or slug flow
		Churn turbulent flow
High gas volume fraction		
	Annular / wall film flow	Annular / wall film flow
	Droplet flow	Droplet flow

ejector-induced co-current down flow system where gas flow rate is primarily controlled by liquid flow rate for a particular gas–liquid mixing height. They found that if liquid velocity increases significantly gas bubbles coalesce which leads to increase in buoyant force and hence they move upward rapidly and change to churn flow, slug flow, etc. However, for co-current down-flow system homogeneous bubbly flow regime is the better selection, otherwise it is quite difficult to move the bubbles in the downward direction. The operating range of the liquid flow rate for bubbly flow was 2.0×10^{-4}–$3.53 \times 10^{-4} m^3/s$ and the corresponding air entrainment rate varied from 0.40×10^{-5} *to* $9.0 \times 10^{-5} m^3/s$.

Zahradnik and Fialova (1996) observed a remarkable change when the superficial gas velocity is increased from 0.04 m s^{-1} (Figure 2). The homogeneous bubble regime is changed and transition bubbling regime starts. In a similar study Rice and Littlefield (1987) also observed that the homogeneous bubbling regime ("ideal bubbly flow") was maintained up to gas superficial velocity equals $0.04\, m\, s^{-1}$. Bakshi et al. (1995) identified that the transition from the homogeneous to the heterogeneous bubbling regime at gas superficial velocity $> 0.04\, m\, s^{-1}$.

BUBBLE SIZE

Under no circumstance fluid jet produce the liquid droplet/bubble of uniform diameter. In the study of jet ejector as gas-liquid contactor, the bubble size is a factor of utmost concern. Bubbles of uniform size are difficult to generate hence “mean bubble size” can be taken as a measure of the quality of the disintegration process. It is also convenient to use mean bubble size in calculations such as multiphase flow and mass transfer processes (Lefebvre, 1989).

In the literature (Lefebvre, 1989) average or mean bubble/particle/droplet diameter, $d_{i,j}$, is defined as

$$d_{i,j} = \frac{\int_0^\infty d^i f(d) dD}{\int_0^\infty d^j f(d)\, dD} \quad (1)$$

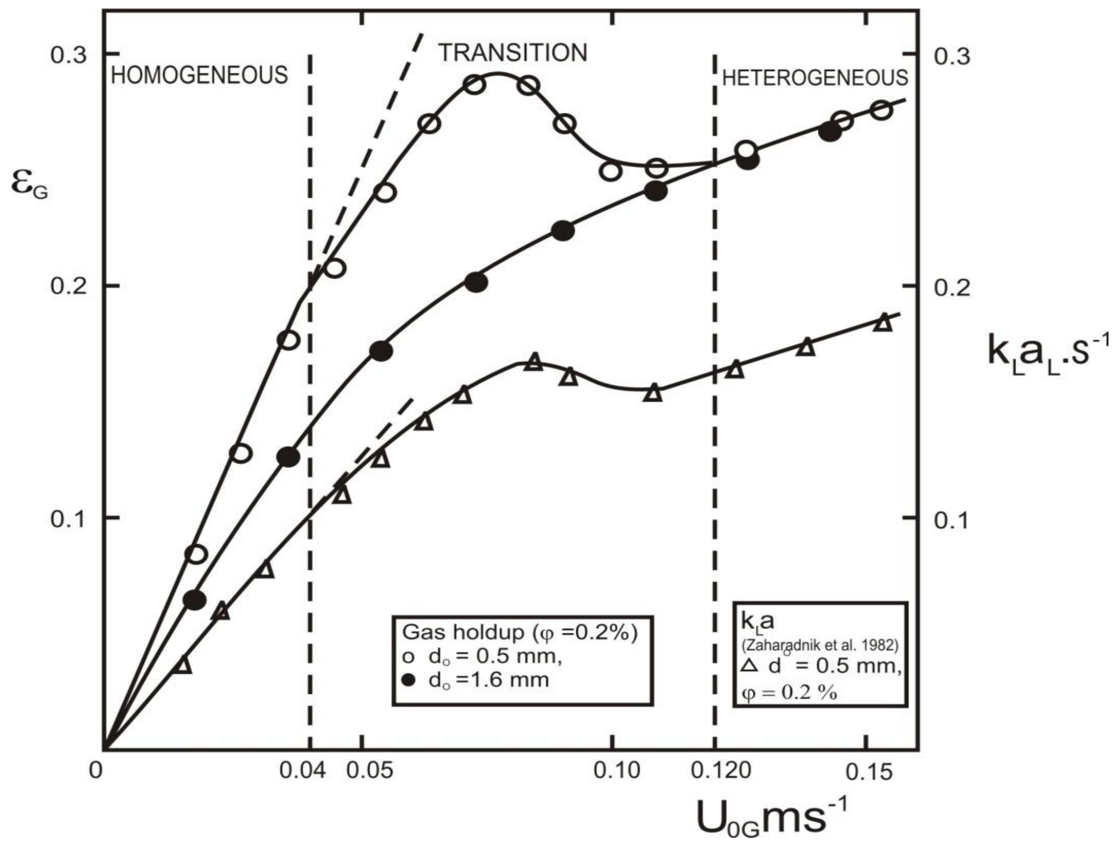

Figure 2. Gas holdup and k_La as function of the superficial gas velocity; (Zahradnik and Fialova, 1996).

where d is the diameter and i and j take any value according to the effects considered (for example, 3 and 2 for SMD, 1 and 0 for arithmetic mean diameter, 2 and 0 for surface mean diameter, 3 and 0 for volume mean diameter). There are many definitions of bubble size which cause confusion. However standard texts (Liu, 2000; Lefebvre, 1989) have summarized different definitions, given by different researchers from time to time in their work. Out of them the most commonly used definition for "jet ejector" is d_{32} . It is defined as the diameter of a sphere that has the same volume/surface area ratio as a bubble of interest. Several methods have been devised to obtain a good estimate of the d_{32}. In the case of jet ejector studies, d_{32}, is found most suitable mean diameter which is also known as Sauter mean diameter (SMD). This diameter has been established most appropriate because it gives the mean value in terms of volume / surface ratio. This relationship is the most suitable because the mass transfer takes place on the surface of droplets/bubbles and the acceleration caused by the drag forces. The drag force is proportional to the projected area of the bubble. Due to this, most of the cases bubble sizes and correlations for jet ejectors are presented in terms of d_{32}. Sauter mean diameter, d_{32}, can be computed using the following equations:

$$d_{32} = \frac{\sum N_i d_i^3}{\sum N_i d_i^2} \quad (2)$$

$$d_{32} = \frac{\int_0^\infty d^3 f(d) dD}{\int_0^\infty d^2 f(d)\, dD} \quad (3)$$

$$d_{32} = \frac{\int_0^\infty \left[d^3 \frac{dN}{dD}\right] dD}{\int_0^\infty \left[d^2 \frac{dN}{dD}\right] dD} \quad (4)$$

d_i is the diameter of a single bubble and N_i is the number of bubbles of diameter d_i.

BUBBLE SIZE DISTRIBUTION

Practically in any gas-liquid two-phase system, gas is either entrained by plunging liquid jets or some time enforced to enter liquid media, do not generally produce dispersion of uniform bubble size at any given operating] condition. On the contrary, the plume of bubble can be regarded as a spectrum of bubble sizes distributed about some arbitrary defined mean value. In the two-phase gas–liquid system, there is simultaneous coalescence and breakup of bubble which are considered to be responsible for the variation in bubble sizes throughout the system.

Arranging the drop size data into a mathematical representation is referred to as drop size distribution. The mathematical representation is most often dependent on the analyzer used. Recently, however, some analyzer manufacturers have allowed the user to select from a list of distribution functions rather than a default drop size distribution function (Schick, 2006).

Accurate knowledge of bubble size distribution as a function of operating conditions of the system is a prerequisite for the fundamental analysis of mass transfer.

There are various distribution functions applied by different investigators. But no single distribution function can correlate all experimental measurement data of bubble sizes. It is also known that none of known distribution functions is universally superior over any other for representing bubble size distribution.

Various distribution functions have been used to fit the existing experimental data. The most commonly used functions are; Rosin-Rammler, Nukiyama-Tanasawa and modified functions such as: upper-limit, logarithmic-normal, and chi-square (Li and Tanki, 1988, 1987). There are also some other distribution functions which were utilized by different researchers like- normal, root normal, gamma distribution, etc.

Because of the natural "cocked hat" shape of typical distribution data, the most logical curves used for representing the data have variations of negative exponentials. That gives an appropriately shaped "tail" in the large diameter end of the curve. However, pure negative exponentials also have an unrealistic finite count at zero diameters, so it must be corrected to give a second tail at the smaller diameter end. This second tail must end with a zero value at zero diameter. Different researchers have modified these correlations using different constants (Dennis, 1966).

The bubble size distributions were measured at different axial positions of the column under steady state of a homogeneous bubbly flow. Various analytical distribution functions were tested by statistical software (SAS) for their adequacy in representing the observed bubble size distributions. It was found that the logarithmic normal distribution provided the most reasonable fittings for all the positions.

The probability function for logarithmic normal distribution $f(\mathrm{D})$ is given by the expression:

$$f(\mathrm{D}) = \frac{dN}{dD} = \frac{1}{\sqrt{2\pi}DS_g}\exp\left[-\frac{1}{2S_g^2}\left(lnD - ln\overline{D}_{ng}\right)^2\right] \quad (5)$$

where $\overline{D}_{ng}$ is the number geometric mean droplet diameter and S_g is the geometric standard deviation.

Azad and Sultan (2006) developed a numerical model for bubble size distribution for both breakage and coalescence in turbulent gas liquid dispersion. Two-step mechanisms are considered for both breakage and coalescence of bubbles. They structured the bubble breakage as the product of the bubble-eddy collision frequency and breakage efficiency in gas-liquid dispersions. Similarly the coalescence function was considered as the product of bubble-bubble collision frequency and coalescence efficiency. They claim that their model is better than previous efforts as their model overcomes several limitations observed such as empirical parameters, narrow range of operating conditions and narrow range of geometries. The predicted bubble size distribution by their model and the experimental data reported in the literature are in good agreement. The percentage of error obtained for the average bubble size was found within $\pm 17\%$. Frank (2005) and Silva et al. (2011) have also done similar work.

Cao and Christensen (2000) studied bubble collapse in a binary solution with simultaneous heat and mass transfer having non-spherically symmetrical condition. They applied a numerical technique to solve the axisymmetric moving boundary problem.

MEASUREMENT TECHNIQUES OF BUBBLE SIZE

Various measurement techniques have been developed and applied with different degrees of success. It is desired that the measurement techniques of bubble properties should be non-interfering and should not create disruption to the flow pattern. An ideal measurement technique should have large range of capability to measure both the spatial and sequential distribution. The measurement technique should be capable to tolerate wide variations in bubble properties at some extreme conditions present in flow in different engineering applications. It should also be able to acquire adequate representative samples so that reasonable measurement accuracy is ensured. (Akafuah, 2009).

For the analysis of the measurement of results of rapid sampling and data processing means are needed. As there is fast breakup and collisions of bubbles taking place, the sampling, data acquisition and processing system must also be fast enough.

The measurement techniques for droplet/bubble sizing may be grouped conveniently into four primary categories:

1. Mechanical methods
2. Electrical methods
3. Optical methods
4. Acoustical methods

Though mechanical and electrical methods are relatively simple and low cost, optical methods are being

Table 2. Optical methods of characterization of two-phase flow.

Categories	Methods	Size range (μm)
Imaging	Photography	$\geq \sim 5$
	Videography	-
	Holography	5 – 1000
Non Imaging	Light-scattering Interferometry	5 – 3000
	Phase-Doppler Anemometry	0.5 – 3000
	Light intensity deconvolution technique	0.2 – 200
	Light scattering technique	10 – 250
	Malvern particle analyzer	1 – 900
	Polarization ratio particle sizer	-
	Intensity ratio method	-
	Phase optical-microwave method	-
	Dual-cylindrical wave laser technique	-

developed and are finding wide range of applications in two-phase flow characterization (Pfeifer, 2010; Vamos, 2010; Kashdan et al., 2007, 2000; Black et al., 1996). An acoustical method has been evaluated for the measurements of fine bubbles. Table 2 summarizes the optical methods of measurement techniques.

CORRELATIONS FOR ENTRAINMENT, BUBBLE DIAMETER, DRAG FORCE AND GAS HOLD UP

The mean bubble size d_{32} is related to the pressure drop, gas ratio and liquid flow. The interfacial area and d_{32} both mainly depend on the local gas hold ups. The gas hold up is influenced by presence or absence of swirl body. Simonin (1959) proposed a quasi-theoretical relationship between the bubble diameter and the entrainment ratio for the air-water system:

$$d_v = 4.3x10^{-3}\left[\frac{Q_A}{Q_w}\right]^{1/3} \tag{6}$$

The bubble volume-equivalent diameter, d_v, is expressed in meter. Equation 6 shows a moderate effect of entrainment ratio and was tested with experimental data by Ciborowski and Bin (1972), giving reasonable agreement.

Ohkawa et al. (1987) studied the flow characteristics and performance of a vertical liquid jet with down comers in an air-water system. Sheng and Irons (1995) made an attempt to model the bubble-breakup phenomenon in which the bubbles greater than critical size was allowed to subdivide into smaller (daughter) bubbles. The critical size was determined from the combined Kelvin-Helmholtz and Rayleigh-Taylor instability theory (Kitscha and Ocamustafaogullari, 1989) as

$$d_{vb} = 6.45u_g\left[\frac{\sigma}{\rho_l g^3}\right]^{0.25} \tag{7}$$

where d_{vb} is the critical (volume-equivalent) diameter of the bubble, σ is the surface tension, g is the gravitational force and u_g is the rise-velocity of the bubble. The local breakup probability was assumed to have a Gaussian distribution. A random number generator is used to determine whether a particular bubble broke up or not. If this was the case, the number and size of the daughter bubbles were also calculated with a random number generator according to a predefined distribution. Further breakup of daughter bubbles was also permitted.

Similarly Baier (2001) developed following equation for calculating bubble diameter produced by jet ejector used in loop reactor.

$$d_{32} = 50.5.\left[\frac{\rho_G}{\rho_L}\right]^{-0.1}.\left[\frac{g}{V_L^2}\right]^{-1/3} \tag{8}$$

where V_L is liquid batch volume.

Evans et al. (1992) discussed the applicability of the familiar model based on a critical Weber number, We, defined by the energy dissipation rate per unit volume of the mixing zone, which enables prediction of the maximum bubble size generated within the mixing zone at the top of a plunging liquid jet bubble column. A final expression for the maximum stable bubble diameter is given by

$$d_{b,max} = {\rho_L}^{-1/5}E^{-2/5}\left[\frac{We_c\,\sigma}{2}\right]^{3/5} \tag{9}$$

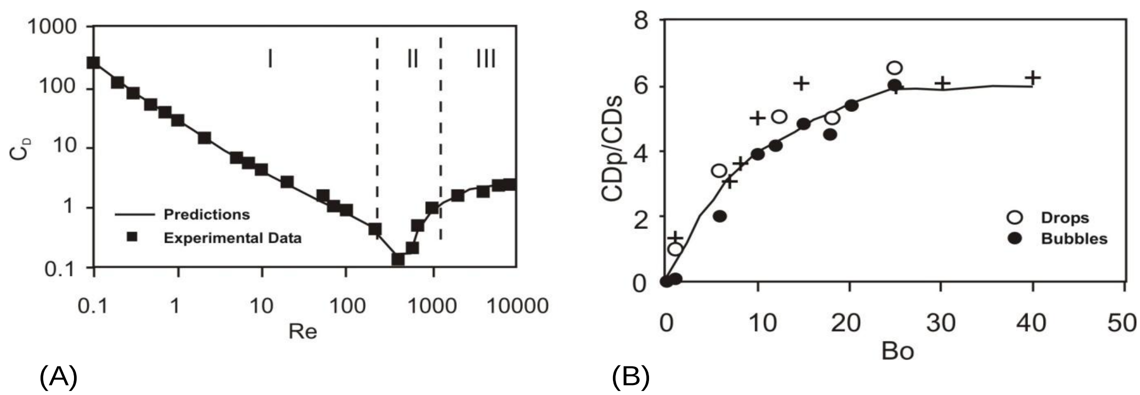

Figure 3. (A) The dependence of drag coefficient on Reynolds number for the deformable particles. (B) Dependence of the drag coefficient on Bond numbers (Bo) for the deformable particle (Ceylan et al., 2001).

where E is the specific energy dissipation rate (per unit volume). Ogawa et al. (1983) gave a final expression to calculate size of bubble for two different sections, calm and spouting, in upward flow bubble column- followed by liquid jet ejector.

Ceylan et al. (2001) proposed the relationships to predict drag coefficient which is applicable for the solid spherical or cylindrical particles (in the range of 0.1≤ *Re* ≤10^6) and for the deformable particles (drops and bubbles in the range of 0.1 ≤ *Re* ≤10^4). They presented their data with respect to Reynolds number and Bond number. Bond number is defined as

$$B_0 = \frac{(\rho L - \rho G)L^2 g}{\sigma} = \frac{\text{gravitational force}}{\text{surface tension force}} \qquad (10)$$

The predicted coefficients were in good agreement with the experimental data given in the literature (Figure 3).

Havelka et al. (1997) studied the effect of swirl, number of nozzles and aspect ratio on gas suction rate and gas hold up in the ejector. They observed that multi nozzle having 4 nozzles, pitch 9.2 mm (ratio of pitch to nozzle diameter = 1.84) and aspect ratio 5, yielded higher value of gas suction rate then single orifice nozzle. Their results are in good agreement with findings of Panchal et al. (1991) who observed that in absence of swirl, multi nozzles yield higher suction rate at optimum pitch $l_d = 2D_{N.}$

Mandal et al. (2005a, b) have also investigated gas holdup, bubble size distribution and interfacial area. They found that bubble sizes have a logarithmic-normal probability distribution for any axial positions of the column. They compared geometric interfacial area obtained from Sauter mean bubble diameters and overall gas holdup with the interfacial area obtained by chemical method. Mandal et al. (2004) have also done similar study for non-Newtonian liquid.

Zahradnik and Fialova (1996) as well Mandal et al. (2003) compared mixing data obtained by them with corresponding dependences of gas holdup and $k_L a$ on the superficial gas velocity.

FACTORS AFFECTING THE BUBBLE SIZE

Baier (2001) observed no significant differences of the mean bubble size (d_{32}) by changing ejector configuration and power input within the operating conditions used in the experiment (Figure 4).

Their findings are in good agreement with other published data (Pawelczyk and Pindur, 1999; Dutta and Raghavan, 1987). Havelka et al. (1997) studied up-flow ejector loop reactor and observed axially and radially uniform bubble size distributions. Bin (1993) has extensively reviewed a large number of studies carried out on plunging liquid jet systems. For the air/water system and found that the secondary bubbles were formed very quickly and had diameters of about 4 mm, practically independent of the jet velocity and the nozzle diameter.

Baier (2001) studied the effect of electrolyte solutions on the average bubble size and observed that electrolyte solution have 10 time smaller bubbles than pure water (Figure 5). This led to a strong increase of both $k_L a$ and the gas holdup. The estimated specific surface area considering mean bubble diameter $(d_{32}) \sim 700$ μm is about 1500 and $6000\ m^{-1}$.

There are studies on the effect of gas density and operating pressure on average diameter. Baier (2001) found that system pressure and molecular weight of carrier gas have a significant influence on the bubble size. With increasing pressure and molecular weight of the gas component the Sauter bubble diameter decreases and the bubble size distribution becomes more narrow (Figures 6 and 7). A strong correlation between the gas density and the Sauter bubble diameter can be identified. The Sauter bubble diameter decreases with increasing gas density, that is, the influence of the

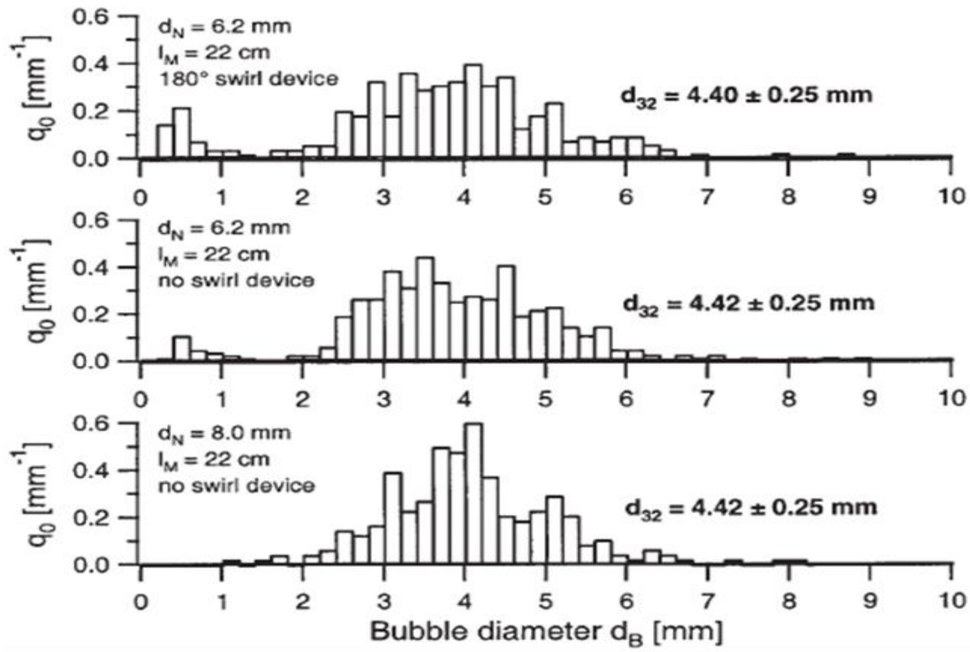

Figure 4. Bubble size distributions at different reaction mixer configurations (Baier, 2001).

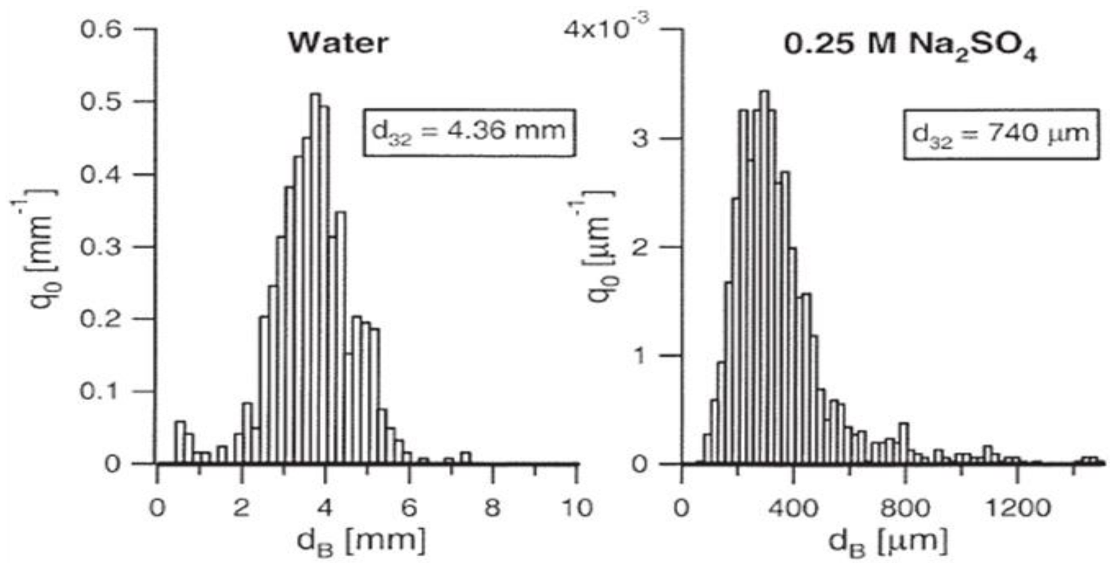

Figure 5. Characteristic bubble size distributions in water and in the 0.25 M Na_2SO_4 solution (Baier, 2001).

system pressure and the gas type can be fully attributed to changes of the gas density. In other words: If different gases of the same density are used, comparable bubble size distributions and Sauter diameters are obtained.

Zheng et al. (2010) used PIV to measure local bubble size distribution, gas–liquid interfacial area and gas holdup in an up-flow ejector, based on the water-air system with different liquid and gas flow rates under the presence/absence of the swirl body. They observed there is the formation of "bubble chain" in the ejector with swirl. The mean bubble sizes in the absence of swirl body are smaller than that in the presence of swirl under different operating conditions. The gas holdups and interfacial area are larger without swirl than those with swirl. Similar conclusions are also presented by Baier (2001).

Literature review suggests that very little work has been carried out on gas-liquid mass transfer with chemical reaction using jet ejector type of contactors.

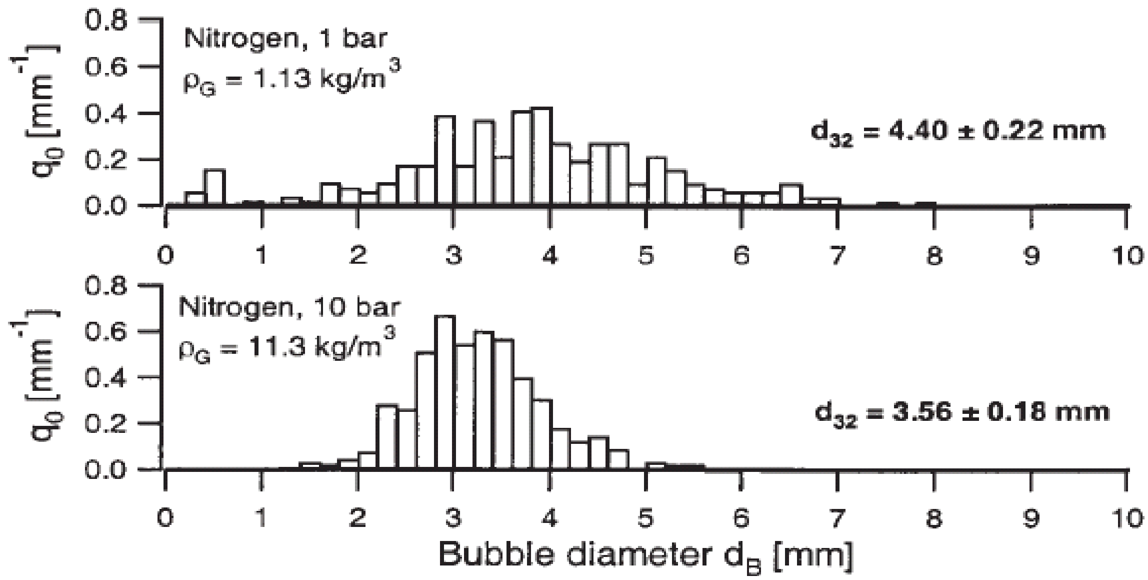

Figure 6. Bubble size distributions with nitrogen at different pressures (Baier, 2001).

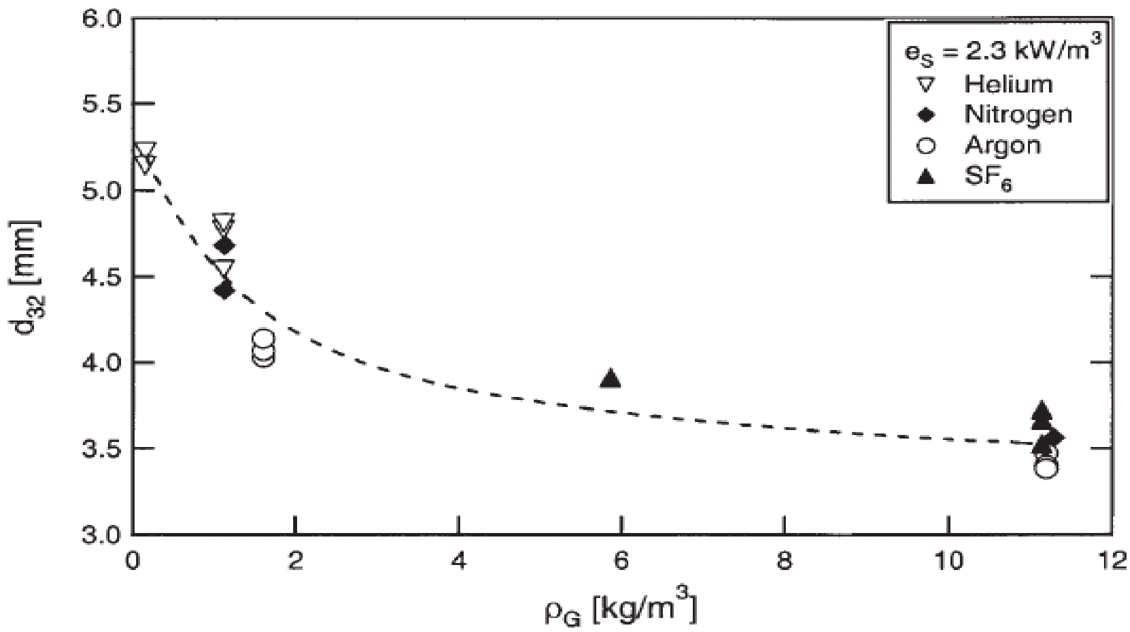

Figure 7. d_{32} versus the gas density (Baier, 2001).

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