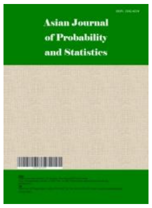

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Tornumonkpe Distribution: Statistical Properties and Goodness of Fit

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

A new sole parameter probability distribution named the Tornumonkpe distribution has been derived in this paper. The new model is a blend of gamma $(2,\theta)$ and gamma$(3,\theta)$ distributions. The shape of its density for different values of the parameter has been shown. The mathematical expression for the moment generating function, the first three raw moments, the second and third moments about the mean, the distribution of order statistics, coefficient of variation and coefficient of skewness has been given. The parameter of the new distribution was estimated using the method of maximum likelihood. The goodness of fit of the Tornumonkpe distribution was established by fitting the distribution to three real life data sets. Using -2ln*L*, Bayesian Information Criterion (BIC), and Akaike Information Criterion(AIC) as criterial for selecting the best fitting model, it was revealed that the new distribution outperforms the one parameter exponential, Shanker and Amarendra distributions for the data sets used.

Keywords: Goodness of fit, entropy; probability distribution; maximum likelihood and statistical properties.

1 Introduction

The study of the probability distribution of random variables is imperative in mathematical statistics; especially, in parametric statistics where random variables are assumed to follow certain probability distributions.

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Probability distributions offer most of the required mathematical foundations for the study of random processes in statistics and thus, considered an integral and indispensable part of statistics, since most statistical data analysis are predicated on an assumed probabilistic distribution for a quality output. When a probabilistic model gives a good fit to a given data set, inferences made about the population parameter based on the output of such analysis are more reliable. There exist in statistical literature a lot of discrete and continuous probability distributions. On the other hand, a lot of studies in probability theory have revealed that regardless of the existence of numerous classical distributions for modelling real life data, the problem of lack of fit and flexibility of the distributions still persist, especially, in the modeling of real life data from various areas of study such as insurance, engineering, biological sciences, finance, medical science and so on Ogutunde [1]

To address these drawbacks, various approaches have been adopted by statisticians. Lately, modifying an existing probability distribution using various generalized families have been proposed. For example, the performance of an existing distribution could be bettered by using generalised families of distributions such as the generalization family developed by Marshall and Olkin [2], Beta Generalised (Beta-G) family of distributions derived by Eugene et al., [3] and exponentiation method given by Gupta et al., [4] amongst others. According to Ogutunde [1], by generalizing an existing distribution, a new distribution with extra shape parameter(s) is obtained. The additional parameter could make the new compound distribution more flexible.

In this light, Ghitany [5] modified the Pareto type 1 using the Marshall-Olkin family of distributions. Similary, the Gamma distribution has been generalized by Ristic [6] using Marshall-Olkin families. The modification of the Uniform distribution has been proposed by Jose and Krishna [7] using Marshall-Olkin family of distribution. Furthermore, other distributions have been generalized using the Beta-generalized family, these distributions include: The Beta-Pareto distribution by Akinsete et al., [8], the Beta-Gompertz probability distribution given by Jafari et al., [9], the Beta-Laplace distribution by Nadarajah [10]; Beta-Nakagami probability distribution by Shittu and Adepoju [11]. Nadarajah and Kotz [10], have also derived some essential statistical properties of the Beta-Gumbel distribution. By generalization, additional parameters are introduced into an existing distribution. The new parameters enhance the flexibility of the new distribution so formed.

Other novel distributions could also be derived by mixing two or more classical distributions to get an entirely new distribution. The Shanker, Akash, Negative binomial and Lindley distributions are some examples of mixed models. It has been shown by Shanker in [12] that the Shanker distribution is more flexible than the classical exponential distribution whose hazard rate is constant. The aim of this study is to create a new distribution that is tractable and flexible in modelling duration data sets from fields of study such as Economics, finance, Biology and Engineering. Consequently, the Tornumonkpe distribution is proposed. The new distribution will be suitable for modeling heavily skewed data set.

2 Tornumonkpe Distribution

A continuous random variable, X is said to follow the Tornumonkpe distribution if its PDF is given by:

$$f(x;\,\theta) = \frac{\theta^3}{(\theta^2+2)}(x^2 + x\,\theta)e^{-\theta x} \quad x > 0, \theta > 0 \tag{1}$$

Proof: The distribution in (1) is a two-component additive mixed model comprising two gamma distributions. Let the first distribution be gamma(2, θ) and the second be gamma(3, θ) thus,

$$f_1(x;\,\theta) = \frac{\theta^2 x e^{-\theta x}}{\Gamma(2)} \text{ and } f_2(x;\,\theta) = \frac{\theta^3 x^2 e^{-\theta x}}{\Gamma(3)}$$

Generally, the expression for a two components additive mixed model with parameter θ is given as:

$$f(x;\,\theta) = p_1 f1(x;\,\theta) + (1 - p_1) f2(x;\,\theta) \tag{2}$$

where $p_1 = \frac{\theta^2}{(\theta^2+2)}$ and $p_2 = \frac{2}{(\theta^2+2)}$ are the mixture proportion

From equation (2), equation (1) is obtained as follows:

$$f(x;\theta) = \frac{\theta^4}{(\theta^2+2)} x e^{-\theta x} + \frac{\theta^3 x^2 e^{-\theta x}}{(\theta^2+2)}$$
$$= \frac{\theta^3}{(\theta^2+2)}(x^2 + x\,\theta)e^{-\theta x}$$

By integrating equation (1), the cumulative distribution function (CDF) of the Tornumonkpe distribution is obtained as follows;

$$F(x;\theta) = \left[1 - \left(1 + \frac{\theta^2 x^2 + \theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right] \quad x > 0, \theta > 0 \tag{3}$$

3 Graphs of the Probability Density Function (PDF) of the Tornumonkpe

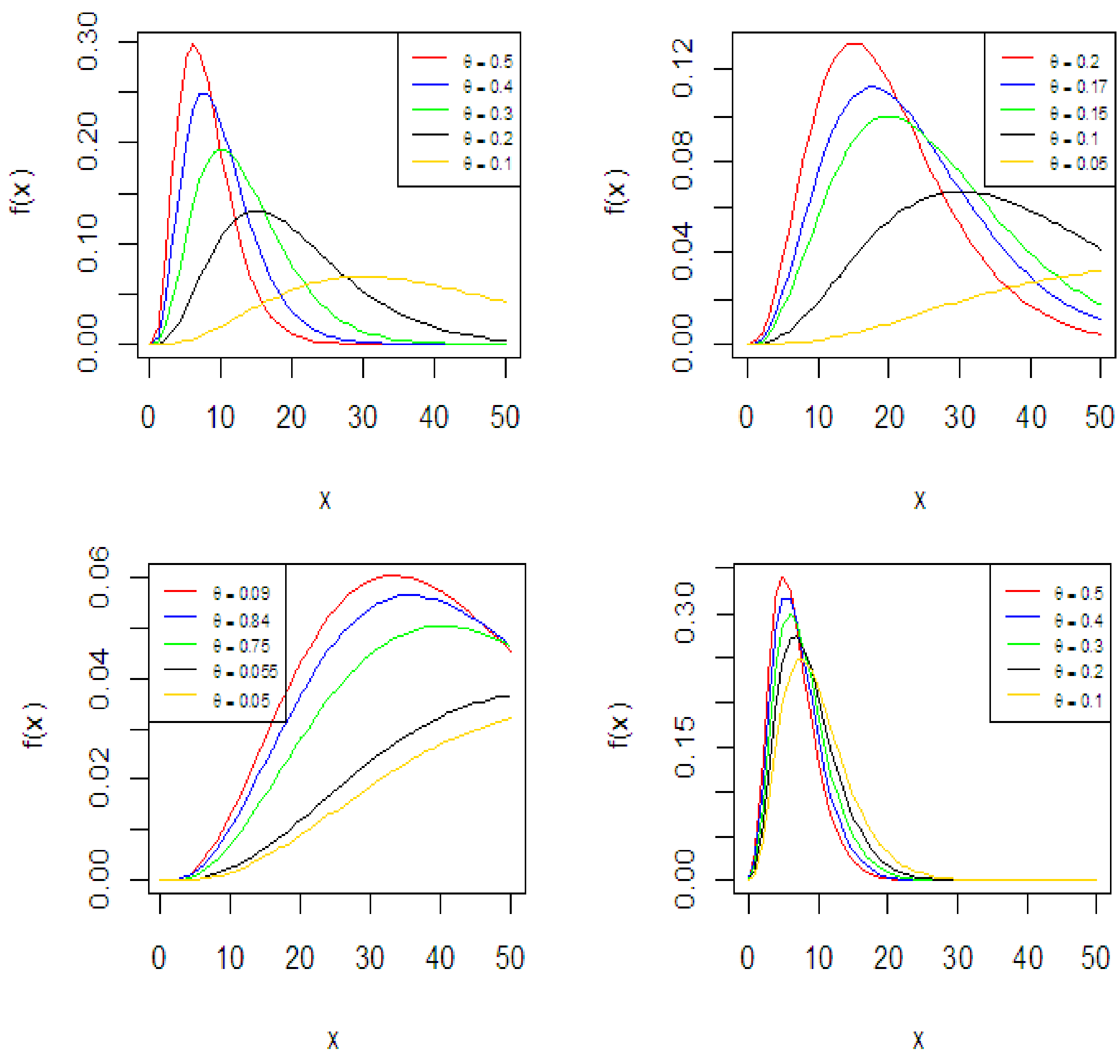

Fig.1. Graph of the pdf of the Tornumonkpe distribution

4 Graphs of Cumulative Distribution Function (CDF) of Tornumonkpe

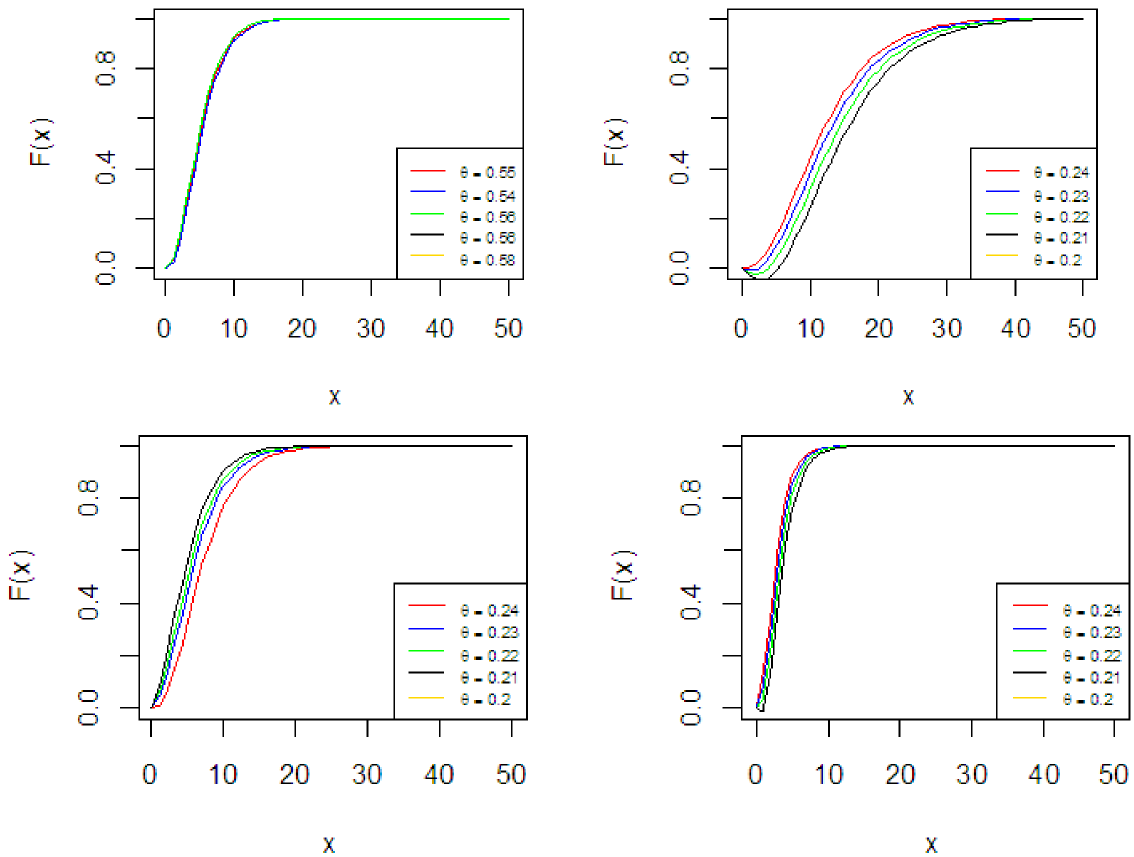

Fig. 2. Graph of the CDF of Tornumonkpe distribution

5 Hazard Rate Function of Tornumonkpe Distribution

In general, the hazard function of a random variable $X$is defined as:

$$h(x) = \frac{f(x)}{1 - F(x)} \tag{4}$$

Where $f(x)$ and $F(x)$ are the Probability density function and the cumulative distribution function respectively. If the random variable X follows the Tornumonkpe distribution then, its hazard function is given as:

$$\begin{aligned} h(x) &= \frac{\frac{\theta^3}{(\theta^2+2)}(x^2+\theta x)e^{-\theta x}}{\left\{1 - \left[1 - \left(1 + \frac{\theta^2 x^2 + \theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]\right\}} \\ &= \frac{\theta^3(x^2+\theta x)e^{-\theta x}}{(\theta^2+2)} \times \left(\frac{(\theta^2+2)}{\theta^2+2+\theta^2 x^2+\theta x(\theta^2+2)e^{-\theta x}}\right) \\ &= \frac{\theta^3(x^2+\theta x)}{\theta^2(x^2+1)+\theta x(\theta^2+2)+2} \end{aligned} \tag{5}$$

6 Graph of Hazard Function of the Tornumonkpe Distribution

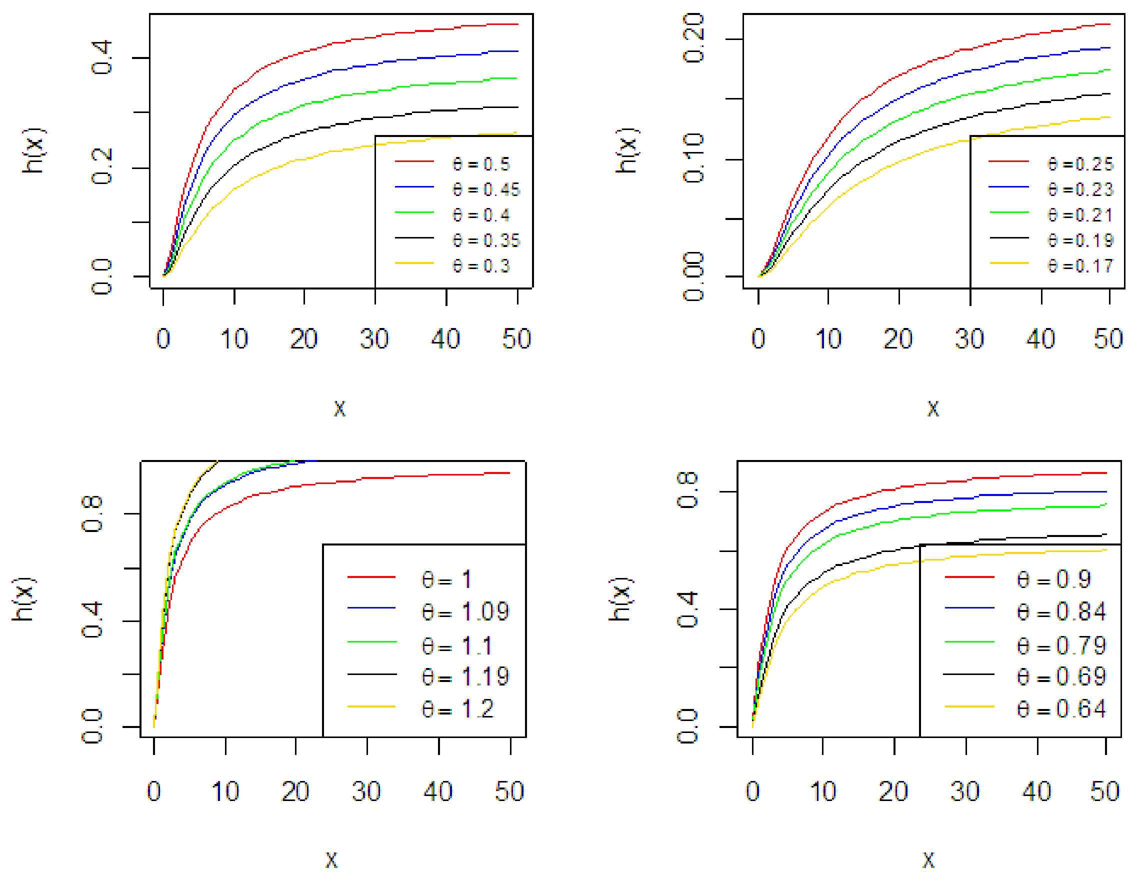

Fig. 3. Graph of the hazard function for Tornumonkp edistribution

7 The Raw Moments of the Tornumonkpe Distribution

The kth raw moment of a random variable X is given as:

$$\mu_k^{'} = E(x^k) = \int_{-\infty}^{\infty} x^k f(x)dx$$

If X follows the Tornumonkpe distribution, its kth raw moment is:

$$\begin{aligned}\mu_k^{'} &= \frac{\theta^3}{(\theta^2+2)}\int_0^{\infty} x^k(x^2+\theta x)dx \\ &= \frac{\theta^3}{(\theta^2+2)}\frac{(k+2)!+\theta^2(k+1)}{\theta^{k+3}} \\ &= \frac{(k+2)!+\theta^2(k+1)}{\theta^k(\theta^2+2)}\end{aligned} \quad (6)$$

Using (6) above we obtain the first four raw moments of the Tornumonkpe distribution as follows:

$$E(X)=\mu_1'=\frac{(6+2\theta^2)}{\theta(\theta^2+2)},\ E(X^2)=\mu_2'=\frac{(6\theta^2+24)}{\theta^2(\theta^2+2)},\ E(X^3)=\mu_3'=\frac{24(\theta^2+5)}{\theta^3(\theta^2+2)}\text{ and}$$
$$E(X^4)=\mu_4'=\frac{120(\theta^2+6)}{\theta^4(\theta^2+2)}$$

8 The Variance of the New Tornumonkpe Distribution

The second moment about the mean which is the variance of the Tornumonkpe distribution was derived using the following equation:

$$\mu_k=E(X-\mu)^k \tag{7}$$

The first central moment is zero while the second central moment is the variance of the distribution given as follows:

$$\begin{aligned}\mu_2 &= \sigma^2=E(X-\mu)^2=E(X^2-2X\mu+\mu^2)\\ &=E(X^2-\mu^2)=E(X^2)-E(X)^2\\ &=\frac{(6\theta^2+24)}{\theta^2(\theta^2+2)}-\left(\frac{(6+2\theta^2)}{\theta(\theta^2+2)}\right)^2\\ &=\frac{6(\theta^2+4)((\theta^2+2)-(6+2\theta^2)(6+2\theta^2)}{\theta^2(\theta^2+2)^2}=\frac{2(\theta^4+6\theta^2+6)}{\theta^2(\theta^2+2)^2}\end{aligned} \tag{8}$$

9 The Third Central Moment of the Tornumonkpe Distribution

Using the relationship between the raw moment and the central we obtain the third central moment as follows:

$$\begin{aligned}\mu_3 &= E(X-\mu)^3\\ &=\mu_3'-3\mu\mu_2'+2\mu^3\\ &=\frac{24(\theta^2+5)}{\theta^3(\theta^2+2)}-3\left(\frac{(6+2\theta^2)}{\theta(\theta^2+2)}\right)\left(\frac{(6\theta^2+24)}{\theta^2(\theta^2+2)}\right)+2\left(\frac{(6+2\theta^2)}{\theta(\theta^2+2)}\right)^3\\ &=\frac{2(2\theta^6+18\theta^4+33\theta^2+24)}{\theta^3(\theta^2+2)^3}\end{aligned} \tag{9}$$

10 The Moment Generating Function (MGF) of the Tornumonkpe Distribution

The moment generating function of any arbitrary random variable X is defined as

$$M_X(t)=E(e^{xt})=\int_{-\infty}^{\infty}e^{xt}f(x)dx$$

If X follows the Tornumonkpe distribution, the MGF of X is

$$\begin{aligned}M_X(t) &=\frac{\theta^3}{(\theta^2+2)}\int_0^\infty e^{xt}(x^2+x\theta)e^{-\theta x}\\ &=\frac{\theta^3}{(\theta^2+2)}\int_0^\infty (x^2+x\theta)e^{-x(\theta-t)}dx\\ &=\left(\frac{\theta^3}{(\theta^2+2)}\right)\left(\frac{\Gamma(3)}{(\theta-t)^3}+\frac{\theta\Gamma(2)}{(\theta-t)^2}\right)=\left(\frac{\theta^3}{(\theta^2+2)}\right)\left[\frac{2!}{(\theta-t)^3}+\frac{\theta}{(\theta-t)^2}\right]\\ &=\left(\frac{\theta^3}{(\theta^2+2)}\right)\left\{\frac{2}{\theta^3}\sum_{k=0}^{\infty}\binom{k+2}{k}\left(\frac{t}{\theta}\right)^k+\frac{\theta}{\theta^2}\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\theta}\right)^k\right\}\end{aligned}$$

$$= \left(\frac{\theta^3}{(\theta^2+2)}\right)\sum_{k=0}^{\infty}\left(\frac{2\,(k+2)(k+1)+\theta^2(k+1)}{\theta^3}\right)\left(\frac{t}{\theta}\right)^k$$

$$= \left(\frac{\theta^3}{(\theta^2+2)}\right)\sum_{k=0}^{\infty}\left\{\frac{2(k+2)(k+1)+\theta^2(k+1)}{(\theta^2+2)}\right\}\left(\frac{t}{\theta}\right)^k$$

The rth moment about the origin is the obtained as the coefficients of $\frac{t^r}{r!}$

$$\mu_r' = \frac{r!\,[(r+2)(r+1)+\theta^2(r+1)]}{\theta^r(\theta^2+2)} \tag{10}$$

11 Distribution of Order Statistics of the Tornumonkpe Distribution

Let $X_1, X_2, X_3 \dots X_P,$ be a random sample from the Tornumonkpe Suppose that the PDF and CDF of X are $f(x)$and $F(x)$ respectively, let the corresponding order statistics obtained from the sample be $X_{1:p} > X_{2:p} > X_{3:p} > \cdots > X_{P:p}$. By the definition, the PDF of the pth order statistics is given by:

$$f_{x:p}(\mathrm{x}) = \frac{p!}{(k-1)!\,(p-k)!}\sum_{i=o}^{p}\binom{p-k}{i}(-1)^i\big(F(x)\big)^{k-1+i}f(x)$$

where

$$F(x;\theta) = \left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]$$

and

$$f(x) = \frac{\theta^3}{(\theta^2+2)}(x^2+\theta x)e^{-\theta x}$$

$$f_{x:p}(\mathrm{x}) = \frac{p!}{(k-1)!\,(p-k)!}\sum_{i=o}^{p}\binom{p-k}{i}(-1)^i\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]^{k-1+i} \times \frac{\theta^3}{(\theta^2+2)}(x^2+\theta x)e^{-\theta x}$$

$$= \frac{p!\,\theta^3(x^2+\theta x)e^{-\theta x}}{(k-1)!\,(p-k)!\,(\theta^2+2)}\sum_{i=o}^{p}\binom{p-k}{i}(-1)^i\sum_{i=o}^{m}\binom{m}{j}(-1)^j \times\left(1+\frac{\theta^3x^3+(\alpha+3)(\theta^2x^2+2\theta x)}{(\theta^5+2\alpha+6)}\right)^{j\,j}$$

Where, m= $k-1+i$.

$$= \sum_{i=o}^{p}\sum_{i=o}^{m}\sum_{n=0}^{k}\binom{p-k}{i}\binom{m}{j}(-1)^{i+j}\frac{p!\,\theta^3(x^2+\theta x)\,e^{-\theta x}}{(k-1)!\,(p-k)!\,(\theta^2+2)} \times\left(1+\frac{\theta^3x^3+(\alpha+3)(\theta^2x^2+2\theta x)}{(\theta^5+2\alpha+6)}\right)^j \tag{11}$$

12 Coefficient of Variation of the Tornumonkpe distribution

The coefficient of variation of a random variable is the ratio of its standard deviation to its mean. For a random variable which follows the Tornumonkpe distribution, coefficient of variation is given by:

$$C.V = \frac{\sqrt{E(X-\mu)^2}}{E(X)} = \frac{\sigma}{\mu_1^{'}} = \sqrt{\frac{2(\theta^4+6\theta^2+6)}{\theta^2(\theta^2+2)^2}} \div \frac{6+2\theta^2}{\theta(\theta^2+2)}$$
$$= \frac{\sqrt{2(\theta^4+6\theta^2+6)}}{2(\theta^2+3)} \qquad (12)$$

13 Coefficient of Skewness of the Tornumonkpe Distribution

Skewness is the measure of the degree to which a statistical distribution deviates from the normal distribution. It is a measure of the level of asymmetry. Mathematically, coefficient of Skewness computed as follows:

$$\beta_3 = \frac{E(X-\mu)^3}{\sigma^3}$$
$$\beta_3 = \frac{E(X-\mu)^3}{\sigma^3} = \frac{\mu_3}{((\mu_2)^{1/2})^3} = \frac{2(2\theta^6+18\theta^4+33\theta^2+24)}{\theta^3(\theta^2+2)^3} \div \left[\frac{2(\theta^4+6\theta^2+6)}{\theta^2(\theta^2+2)^2}\right]^{3/2}$$
$$= \frac{2(2\theta^6+18\theta^4+33\theta^2+24)}{2(\theta^4+6\theta^2+6)^{3/2}} \qquad (13)$$

14 The Renyi's Entropy of the Tornumonkpe Distribution

The entropy of a random variable is the average amount of uncertainty or randomness intrinsic in the realizations of the random variable. For any continuous random variable $\boldsymbol{X}$,the Renyi's entropy of order $\boldsymbol{\delta}$ ($\boldsymbol{T}\ (\boldsymbol{\delta})$) is defined as:

$$T(\delta) = \frac{1}{1-\delta} log\left\{\int_0^\infty f^\delta(x)dx\right\} \qquad (14)$$

Where $f(x)$ is the probability density function of X and $\delta \neq 1$

We obtained the Renyi's entropy of the Tornumonkpe distribution using equation (14) as follows:

$$T(\delta) = \frac{1}{1-\delta} log\left\{\int_0^\infty \left(\frac{\theta^3(x^2+\theta x)e^{-x\theta}}{\theta^2+2}\right)^\delta dx\right\}$$
$$= \frac{1}{1-\delta} log\left\{\left(\frac{\theta^3}{\theta^2+2}\right)^\delta \int_0^\infty \left((x^2+\theta x)e^{-x\theta}\right)^\delta e^{-x\theta\delta}dx\right\}$$
$$= \frac{1}{1-\delta} log\left\{\left(\frac{\theta^3}{\theta^2+2}\right)^\delta \int_0^\infty \left[\theta x\left(\frac{x}{\theta}+1\right)\right]^\delta dx\right\}$$
$$= \frac{1}{1-\delta} log\left\{\left(\frac{\theta^3}{\theta^2+2}\right)^\delta \theta^\delta \int_0^\infty x^\delta \sum_{i=0}^\infty \binom{\delta}{i}\left(\frac{x}{\theta}\right)^i e^{-x\theta\delta}dx\right\}$$
$$= \frac{1}{1-\delta} log\left\{\left(\frac{\theta^3}{\theta^2+2}\right)^\delta \theta^{\delta-i} \sum_{i=0}^\infty \binom{\delta}{i} \int_0^\infty x^{\delta+i} e^{-x\theta\delta}\, dx\right\}$$
$$= \frac{1}{1-\delta} log\left\{\frac{\theta^{4\delta-i}}{(\theta^2+2)^\delta} \sum_{i=0}^\infty \binom{\delta}{i} \frac{\Gamma(\delta+i+1)}{(\theta\delta)^{\delta+i+1}}\right\} \qquad (15)$$

15 Goodness of fit of the Tornumonkpe Distribution

To test the goodness of fit of the Tornumonkpe distribution, the distribution was fitted to three data sets, the performance of the distribution was matched with the performance of Shanker, exponential, Lindley, Akash, Amarendra, and Sujatha distributions for the data sets.

Data Set 1

The first data set represent the length of time (in years) that 81 randomly selected Nigerian graduates stayed without job before been employed by the universal basic education commission in 2011.

2,5,7,5,6,7,7,6,6,9,9,6,6,7,5,4,5,2,9,8,5,9,6,6,7,2,8,3,6,6,2,8,5,7,4,5,6,8,8,9,3,7,6,2,6,8,9,7,6,6,9,5,9,5,5,5,3,9,8,6, 6,6,7,9,4,4,6,9,7,8,8,9,4,6,3,5,4,7,6,6,5

Data Set 2

The second data is the relief times in minutes of patients taking painkiller. The data set was given by Gross and Clark (1975) in Shanker [12], it has also been used by Shanker [12] to fit the Amerandra distribution. The data set consists of twenty (20) observations given as follows:

1.1,1.4,1.3,1.7,1.9,1.8,1.6,2.2,1.7, 2.7, 4.1,1.8,1.5,1.2,1.4, 3,1.7, 2.3,1.6, 2

Data Set 3

The third data set is the strength data of glass of the aircraft window given by Fuller et al 1994) Studied by Shanker [6]. The data set is given below

18.83,20.80,21.657,23.03,23.23,24.05,24.321,25.50,25.52,25.80,26.69,26.77,26.78,27.05, 27.67,29.90,31.11,33.20,33.73,33.76,33.89,34.76,35.75,35.91,36.98,37.08,37.09,39.58, 44.045,45.29,45.381

Table 1. Goodness of The Tornumonkpe Distribution for data set1

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Tornumonkpe	0.4723	381.91	383.91	383.82	383.96	1
Exponential	0.1640	454.91	456.91	459.30	456.9	5
Shanker	0.3085	408.92	410.93	410.83	410.97	4
Lindley	0.2910	418.58	420.58	420.48	420.63	
Sujatha	0.4404	392.39	394.386	394.30	394.44	3
Akash	0.4605	388.61	390.61	390.52	390.57	2

Table 2. Goodness of fit of the Tornumonkpe Distribution for data set 2

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Exponential	0.526	65.7	67.7	68.7	67.9	7
Tornumonkpe	1.314	50.21	52.216	51.51	52.43	1
Shanker	0.804	59.78	61.783	61.081		5
Amerandra	1.481	55.64	57.64	58.63	57.86	2
Sujatha	1.137	57.50	59.50.	60.49	59.72	3
Akash	1.157	59.50	61.70	61.72	61.72	4
Lindley	0.816	60.50	62.50	63.49	62.72	6

Table 3. Goodness of fit of the Tornumonkpe Distribution for data set3

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Exponential	0.032	274.53	276.53	277.96	276.67	6
Tornumonkpe	0.097	240.55	242.553	242.041	242.69	1
Shanker	0.065	252.30	254.30	255.80	254.50	4
Sujatha	0.096	241.503	243.503	244.943	243.643	3
Akash	0.0971	240.68	242.682	244.10	242.80	2
Lindley	0.06293	252.993	255.993	257.423	256.133z	5

16 Discussion and Conclusion

In this paper, a novel probability distribution with one parameter has been derived. The density function of the new distribution is a two component additive mixture of gamma distribution. The graphs of the density function in Fig. 1 reveal that the distribution could be used to model heavily skewed data sets. Fig. 3 shows that the hazard function of the distribution is increasing. Some statistical properties of the distribution such as the distribution of order statistics, crude and raw moments, Renyi's entropy and moment generating function have been derived. The parameter of the distribution was estimated using the method of maximum likelihood. The performance of the new model was determined by fitting it to three real data set. Using goodness of fit criteria such as BIC, AIC, AICC and -2inL it is shown in Tables 1, 2 and 3 that the Tornumonkpe distribution gives a better fit to the data sets used in this work compared with the competing distributions.

Note

The paper is dedicated to my revered Uncle and my Beloved sister, Mr and Mrs Tornubari Monkpe.

Competing Interests

Author has declared that no competing interests exist.

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