British Journal of Mathematics & Computer Science
4(1): 120-132, 2014

SCIENCEDOMAIN *international*
www.sciencedomain.org

Fuzzy Cost Computations of M/M/1 and M/G/1 Queueing Models

M. Jeeva[1] and E. Rathnakumari[1*]

[1]*Department of Mathematics, Ethiraj College for Women, Chennai, India.*

Authors' contributions

This work was carried out in collaboration between all authors. Author MJ suggested the basic idea and designed the paper. Author ER wrote the first draft of the manuscript, derived the model and calculated hypothetical example. Both the authors read and approved the final manuscript and did the corrections suggested by the reviewers.

Research Article

Received: 03 April 2013
Accepted: 08 July 2013
Published: 16th October 2013

Abstract

In this paper two models of planning queuing system and its effect on the cost of the each system by using two fuzzy queuing models of M/M/1 and M/G/1 are studied. These two fuzzy queuing models based on the cost of each model are compared and fuzzy ranking methods are used to select the optimal model due to the resulted complexity. Fuzzy queuing is more practical and realistic than deterministic queuing models. The basic idea is to transform a fuzzy queuing cost problem to a family of conventional crisp queue cost problem by applying the α-cut approach and Zadeh's extension principle. A set of parametric nonlinear programs are developed to calculate the lower and upper bound of the minimal expected total cost per unit time at α, through which the membership function of the total cost is constructed. Numerical example is illustrated to check the validity of the proposed method.

Keywords: α-cut, membership function, total cost function, Centroid ranking method.

1 Introduction

In this modern world, it is well known that manpower is inevitable in spite of existence of advanced technology. Manpower planning is a device with which an attempt is made to match the supply of people with the demand in the form of jobs available in an organisation so that the cost incurred is optimal.

**Corresponding author: rrathnakumari11@gmail.com;*

Queuing decision problem play an important role in the queuing system design that involves one or many decision such as number of servers at a service facility, the efficiency of the servers. A queuing cost based decision model is to determine a suitable service rate such that the sum of the cost of offering the service and cost of delay in offering the service is minimized.

Crisp models of M/M/1 and M/G/1 are studied in the works of Hiler and Liberman in [1] and Taha [2]. Fuzzy queuing models have been described by such researchers like Li and Lee [3], Buckley [4,5], Negi and Lee [6]. Chen [7,8] analysed fuzzy queuing using Zadeh's extension principle. Kao et al. [9] constructed membership function of the system characteristic for fuzzy queues using parametric linear programming. Pardo and Fuente considered optimal selection of service rate for a infinite source and optimizing priority queuing discipline under fuzzy environment [10,11]. Barak and Fallahnezhad [12] studied cost analysis of fuzzy queuing system.

In queuing theory it is usually assumed that the time between the two consecutive arrivals and the servicing time follows a special probability distribution. However, in the real world, this type of information is obtained using qualitative data and expressed by words like quick, medium and slow rather than the probability distribution. Hence fuzzy queuing models are more realistic and practical than classical ones.

In this paper, fuzzy cost computations M/M/1 and M/G/1 models are considered and, the cost measure of each model is evaluated. Here arrival rate, service rate, system cost are taken as fuzzy numbers. According to experts experience to express the uncertain condition in the system completely. Since fuzzy variables capture measurement uncertainties as part of experimental data, they are more attuned to reality than crisp variables. Therefore fuzzy approach is better than crisp approach. Further fuzzy ranking is used to compare total cost of two models. Obviously when the cost coefficients, arrival rate service rate are fuzzy, the minimal expected total cost per unit time will be fuzzy. Therefore the minimal expected total cost should be described by the membership function rather than by a crisp value.

Here mathematical non-linear parametric programming approach for the queuing decision problem by the basic idea of Zadeh's extension principle and α cut representation is developed. A set of non linear programming problems are formulated to calculate the upper and lower bound of α cut of the minimal expected total cost and consequently membership function of the minimal expected total cost is derived.

2 Preliminaries

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let $\tilde{A}$ be a fuzzy number, whose membership function $f_{\tilde{A}}(x)$ can generally be defined as [13].

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x), a \le x \le b \\ \omega, b \le x \le c \\ f_{\tilde{A}}^{R}(x), c \le x \le d \end{cases}$$

where $0 < \omega \le 1$ is a constant, $f_{\tilde{A}}^{L} : [a,b] \to [0,\omega]$ and $f_{\tilde{A}}^{R} : [c,d] \to [0,\omega]$ are two strictly monotonical and continuous mappings from R to a closed interval [0, ω]. If ω is 1 then $\tilde{A}$ is a normal fuzzy number; otherwise, it is said to be a non-normal fuzzy number. If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, then $\tilde{A}$ is referred to as a trapezoidal fuzzy number and is usually denoted by $\tilde{A}$ = [a,b,c,d].

A fuzzy set A on R is convex if and only if $A(\lambda x_1 + (1 - \lambda)x_2) \le \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$ where min denotes minimum operator.

Centroid ranking formula for trapezoid fuzzy number $\tilde{A}$ = [a, b, c, d] is given by [14]

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right]$$

$$\tilde{y}_0(\tilde{A}) = \frac{1}{3}\left[1 + \frac{c - b}{(d + c) - (a + b)}\right]$$

$$\text{Rank of } \tilde{A} = \sqrt{(\tilde{x}_0(\tilde{A}))^2 + (\tilde{y}_0(\tilde{A}))^2}$$

3 Total Cost Function of Fuzzy M/M/1 and M/G/1 Queuing Decision Problem

Consider an (FM/FM/1): (∞/FCFS) and (FM/FG/1) : (∞/FCFS)) queuing models in which customer arrive at the service facility at rate $\tilde{\lambda}$ and at service rate $\tilde{\mu}$, the cost of service per unit time is $\tilde{C}_1$, the cost waiting per customer per unit time is $\tilde{C}_2$ for both the models are fuzzy numbers. Let the service rate be $\tilde{\mu}$ for the first model and $\tilde{\mu}_1$ for the later model. Total expected cost for the first model can be computed as follows:

Let $\eta_{\tilde{\lambda}}^{(x)}$, $\eta_{\tilde{\mu}}^{(y)}$, $\eta_{\tilde{C}_1}^{(u)}$ and $\eta_{\tilde{C}_2}^{(v)}$ denoted the membership functions of $\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{C}_1$, and $\tilde{C}_2$ respectively. We have the following fuzzy sets.

$$\tilde{\lambda} = \{(x, \eta_{\tilde{\lambda}}^{(x)}) / x \in X\} \tag{1a}$$

$$\tilde{\mu} = \{(y, \eta_{\tilde{\mu}}^{(y)}) / y \in Y\} \tag{1b}$$

$$\tilde{C}_1 = \{(u, \eta_{\tilde{C}_1}^{(u)}) / u \in U\} \tag{1c}$$

$$\tilde{C}_2 = \{(v, \eta_{\tilde{C}_2}^{(v)}) / v \in V\} \tag{1d}$$

Where X, Y, C_1 and C_2 are the crisp universal sets of arrival service and cost coefficient. Let f(x, y, u, v) denote the system characteristics of interest. Since $\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{C}_1$, $\tilde{C}_2$ are fuzzy numbers, $f(\tilde{\lambda}, \tilde{\mu}, \tilde{C}_1, \tilde{C}_2)$ is also a fuzzy number. Following Zadeh's extension principle [15] the membership function of expected total cost is defined as

$$\eta_{f(\tilde{\lambda},\tilde{\mu},C_1,C_2)}(z) = \sup\min\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y), \eta_{\tilde{C}_1}(u), \eta_{\tilde{C}_2}(v) / \quad z = f(x,y,u,v) \tag{2}$$

The minimal expected total cost of a crisp queuing system is given by

$$E(C) = C_1 + LC_2 \tag{3}$$

The membership function of the minimal cost is

$$\eta_{E(\tilde{C})}^{(z)} = \sup\min\{\eta_{\tilde{\lambda}}^{(x)}, \eta_{\tilde{\mu}}^{(y)}, \eta_{\tilde{C}_1}^{(u)}, \eta_{\tilde{C}_2}^{(v)} / z = u + Lv\} \tag{4}$$

In this paper we approach the representation problem using a mathematical programming technique, parametric NLPs are developed to find α cut of $f(\tilde{\lambda}, \tilde{\mu}, \tilde{C}_1, \tilde{C}_2)$ based on the extension principle.

4 Solution Procedure

Definitions for the α cuts of $\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{C}_1$, and $\tilde{C}_2$ as crisp intervals are as follows.

$$\lambda(\alpha) = [x_\alpha^L, x_\alpha^U] = [\min\{x / \eta_{\tilde{\lambda}}(x) \geq \alpha\} \max\{x / \eta_{\tilde{\lambda}}(x) \geq \alpha\}] \tag{5a}$$

$$\mu(\alpha) = [y_\alpha^L, y_\alpha^U] = [\min\{y / \eta_{\tilde{\mu}}(y) \geq \alpha\} \max\{y / \eta_{\tilde{\mu}}(y) \geq \alpha\}] \tag{5b}$$

$$u(\alpha) = [u_\alpha^L, u_\alpha^U] = [\min\{u / \eta_{\tilde{C}_1}(u) \geq \alpha\} \max\{u / \eta_{\tilde{C}_1}(u) \geq \alpha\}] \tag{5c}$$

$$v(\alpha) = [v_\alpha^L, v_\alpha^U] = [\min\{v / \eta_{\tilde{C}_2}(v) \geq \alpha\} \max\{v / \eta_{\tilde{C}_2}(v) \geq \alpha\}] \tag{5d}$$

As a result, the bound of these intervals can be described as functions of α and can be obtained as

$$x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha) \qquad x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$$
$$y_\alpha^L = \min \eta_{\tilde{\mu}}^{-1}(\alpha) \qquad y_\alpha^U = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$$
$$u_\alpha^L = \min \eta_{\tilde{C}_1}^{-1}(\alpha) \qquad u_\alpha^U = \max \eta_{\tilde{C}_2}^{-1}(\alpha)$$
$$v_\alpha^L = \min \eta_{\tilde{C}_2}^{-1}(\alpha) \qquad v_\alpha^U = \max \eta_{\tilde{C}_2}^{-1}(\alpha)$$

Therefore α- cuts to construct its membership function is used. Since the membership function in (4) is parameterized by α.

Using Zadeh's extension principle, $\eta_{E(\tilde{C})}$ is minimum of $\eta_{\tilde{\lambda}}^{(x)}$, $\eta_{\tilde{\mu}}^{(y)}$, $\eta_{\tilde{C}_1}^{(y)}$, and $\eta_{\tilde{C}_2}^{(y)}$. To derive the $\eta_{E(\tilde{C})}(z) = \alpha$ we need at least one of the following cases to hold such that satisfies $\eta_{E(\tilde{C})}(z) = \alpha$

Case 1: $\eta_{\tilde{\lambda}}^{(x)} = \alpha, \eta_{\tilde{\mu}}^{(y)} \geq \alpha, \eta_{\tilde{C}_1}^{(u)} \geq \alpha, \eta_{\tilde{C}_2}^{(v)} \geq \alpha$

Case 2: $\eta_{\tilde{\lambda}}^{(x)} \geq \alpha, \eta_{\tilde{\mu}}^{(y)} = \alpha, \eta_{\tilde{C}_1}^{(u)} \geq \alpha, \eta_{\tilde{C}_2}^{(v)} \geq \alpha$

Case 3: $\eta_{\tilde{\lambda}}^{(x)} \geq \alpha, \eta_{\tilde{\mu}}^{(y)} \geq \alpha, \eta_{\tilde{C}_1}^{(u)} = \alpha, \eta_{\tilde{C}_2}^{(v)} \geq \alpha$

Case 4: $\eta_{\tilde{\lambda}}^{(x)} \geq \alpha, \eta_{\tilde{\mu}}^{(y)} \geq \alpha, \eta_{\tilde{C}_1}^{(u)} \geq \alpha, \eta_{\tilde{C}_2}^{(v)} = \alpha$

This can be accomplished using parametric NLP techniques. The NLP to find the lower and upper bounds of the cut of $\eta_{E(\tilde{C})}$ for case 1 are

$$[E(C)]_\alpha^{L_1} = \min[u + Lv] \tag{6a}$$

$$[E(C)]_\alpha^{u_1} = \max[u + Lv] \tag{6b}$$

For case 2 are

$$[E(C)]_\alpha^{L_2} = \min[u + Lv] \tag{6c}$$

$$[E(C)]_\alpha^{U_2} = \max[u + Lv] \tag{6d}$$

For case 3 are

$$[E(C)]_\alpha^{L_3} = \min[u + Lv] \tag{6e}$$

$$[E(C)]_\alpha^{U_3} = \max[u + Lv] \tag{6f}$$

For case 4 are

$$[E(C)]_\alpha^{L_4} = \min[u + Lv] \tag{6g}$$

$$[E(C)]_\alpha^{U_4} = \max[u + Lv] \tag{6h}$$

From the definitions of $\lambda(\alpha), \mu(\alpha), u(\alpha)$ and $v(\alpha)$ in (5a, 5b, 5c, 5d) $x \in \lambda(\alpha), y \in \mu(\alpha), u \in C_1(\alpha)$ and $v \in C_2(\alpha)$ can be replaced by $x \in [x_\alpha^L, x_\alpha^U], y \in [y_\alpha^L, y_\alpha^U], u \in [u_\alpha^L, u_\alpha^U], v \in [v_\alpha^L, v_\alpha^U]$.

The α cut form a nested structure with respect to α (6a,6b,6c,6d,6e,6f,6g,6h). For given $0 < \alpha_2 < \alpha_1 < 1$ we have

$$[x_{\alpha_1}^L, x_{\alpha_1}^U] \subseteq [x_{\alpha_2}^L, x_{\alpha_2}^U]$$
$$[y_{\alpha_1}^L, y_{\alpha_1}^U] \subseteq [y_{\alpha_2}^L, y_{\alpha_2}^U]$$
$$[u_{\alpha_1}^L, u_{\alpha_1}^U] \subseteq [u_{\alpha_2}^L, u_{\alpha_2}^U]$$
$$[v_{\alpha_1}^L, v_{\alpha_1}^U] \subseteq [v_{\alpha_2}^L, v_{\alpha_2}^U]$$

Therefore ((6a), (6c), (6e), (6g)) have the same smallest element and ((6b),(6d),(6f),(6h)) have the same largest element.

To find the lower and upper bounds of E(C),

$$[E(C)]^L = \min[u + Lv] \text{ such that} \tag{7a}$$

$$x_\alpha^L \le x \le x_\alpha^U, y_\alpha^L \le y \le y_\alpha^U, u_\alpha^L \le u \le u_\alpha^U, v_\alpha^L \le v \le v_\alpha^U$$

$$[E(C)]^U = \max[u + Lv] \text{ such that} \tag{7b}$$

$$x_\alpha^L \le x \le x_\alpha^U, y_\alpha^L \le y \le y_\alpha^U, u_\alpha^L \le u \le u_\alpha^U, v_\alpha^L \le v \le v_\alpha^U$$

At least any one of x, y, u, v must hit the boundaries of their α cut satisfying $\eta_{E(\tilde{C})}(z) = \alpha$.

Applying the results of Zimmerman [16] and convexity properties [17], we have

$$[E(C)]_{\alpha_1}^L \ge [E(C)]_{\alpha_2}^L \quad \text{and} \quad [E(C)]_{\alpha_1}^U \ge [E(C)]_{\alpha_2}^U$$

Where $0 < \alpha_2 < \alpha_1 < 1$

In both $[E(C_1)]_\alpha^L$ and $[E(C)]_\alpha^U$ are invertible with respect to, then a left shape function $L(z) = [E(C)_\alpha^L]^{-1}$ and right shape function $R(z) = [E(C)_\alpha^U]^{-1}$ can be derived, such that

$$\eta_{E(\tilde{C})} = \begin{cases} L(z), [E(c)]_{\alpha=0}^{L} \le z \le [E(c)]_{\alpha=1}^{L} \\ 1, [E(c)]_{\alpha=1}^{L} \le z \le [E(c)]_{\alpha=1}^{U} \\ R(z), [E(c)]_{\alpha=1}^{U} \le z \le [E(c)]_{\alpha=0}^{U} \end{cases} \tag{8}$$

In most cases the values $\lambda \in [0,1]$ and $[E(C)]_{\alpha}^{U}$ cannot be solved analytically. Consequently, a closed form membership function for $E(\tilde{C})$ cannot be obtained. However, the numerical solution for $[E(C)]_{\alpha}^{L}$ and $[E(C)]_{\alpha}^{U}$ at different possibility levels can be collected to approximate the shape of L (z) and R (z). Similarly the total expected cost for the (FM/FG/1):(∞/FCFS) model can be computed.

Since the superiority of each model cannot be studied intuitively, to decide in an uncertain environment and to compare the different decisions in the fuzzy environment, ranking methods such as Decooman [18], Liou and Wang [19] , Nakumura [20] , centroid ranking method have been proposed. Centroid ranking method is used in this paper.

5 Numerical Example

Consider a single server Poisson input queue with mean arrival rate $\tilde{\lambda}$ = [0.01, 0.02, 0.03, 0.04] customers per unit time, currently the server works according to an exponential distribution with mean service rate $\tilde{\mu}$ = [0.06, 0.07, 0.08, 0.09] customers per unit time. Management has a training course which will result in an improvement in the service rate. After the completion of the course, it is estimated that mean service rate will increased. The increased service rate is $\tilde{\mu}_1$= [0.11, 0.12, 0.13, 0.14]. It is assumed that now the service rate follows the geometric distribution. The cost of the server is a trapezoidal fuzzy number $\tilde{C}_1$ = [1000, 1200, 1400, 1600] per unit time, waiting time cost of the customer is $\tilde{C}_2$ = [50, 60, 70, 80] per unit time. Let $\tilde{C}_3$ = [10, 20, 30, 40] be the training cost of the server. The management wants to know whether the training is useful.

It is easy to find that

$[x_{\alpha}^{L}, x_{\alpha}^{U}] = [0.01 + \alpha., 0.04 - \alpha]$, $\quad [y_{\alpha}^{L}, y_{\alpha}^{U}] = [0.06 + \alpha, 0.09 - \alpha]$,

$[y_{1\alpha}^{L}, y_{1\alpha}^{U}] = [.011 + \alpha, .014 - \alpha]$, $[u_{\alpha}^{L}, u_{\alpha}^{U}] = [1000 + 200\alpha, 1600 - 200\alpha]$

$[v_\alpha^L , v_\alpha^U] = [50 + 10\alpha , 80 - 10\alpha]$

It is obvious that $x = x_\alpha^L$, $y = y_\alpha^U$ $u = u_\alpha^L$, $v = v_\alpha^L$ the expected total cost attains its minimum value and when $x = x_\alpha^U$, $y = y_\alpha^L$, $u = u_\alpha^U$, $v = v_\alpha^U$ the expected total cost attains its maximum value.

According to (7a) and (7b), the α-cuts of

$$[E(C_1)]_\alpha^L = (1000 + 200\alpha) + (50 + 10\alpha)[(0.09-\alpha)/(0.08-0.02\alpha)]$$

$$[E(C_1)]_\alpha^U = (1600-200\alpha) + (80-10\alpha)[(0.06+\alpha)/(0.02+0.02\alpha)]$$

$$[E(C_2)]_\alpha^L = 1000 + 200\alpha + [\frac{0.01\alpha + 0.01}{0.14 - 0.01\alpha} + \frac{(0.01 + 0.01)^2 + (0.01 + 0.01)^2 (0.86 - 0.01\alpha)}{2(0.14 - 0.01\alpha)(0.013 - 0.02\alpha)} + 10 + 10\alpha$$

$$[E(C_2)]_\alpha^U = 1600 - 200\alpha + [\frac{0.04\alpha - 0.01}{0.11 + 0.01\alpha} + \frac{(0.04 - 0.01)^2 + (0.04 - 0.01)^2 (0.89 - 0.01\alpha)}{2(0.11 + 0.01\alpha)(0.07 + 0.02\alpha)} + 40 - 10\alpha$$

Table 1. The α-cuts of the performance measures of 11 α values

A	$[E(C_1)]_\alpha^L$	$(E(C_1))_\alpha^U$	$(E(C_2))_\alpha^L$	$(E(C_2))_\alpha^U$
0.0	1006.25	1760	1013.69	1677.44
0.1	1027.19	1720.04	1035.20	1654.33
0.2	1048.21	1683.50	1056.71	1631.31
0.3	1069.31	1649.58	1078.26	1608.41
0.4	1090.5	1617.71	1099.84	1585.6
0.5	1111.79	1587.50	1121.45	1562.89
0.6	1133.18	1558.63	1143.09	1540.25
0.7	1154.68	1530.85	1164.75	1517.70
0.8	1176.31	1504	1186.46	1495.23
0.9	1198.08	1477.92	1208.19	1472.82
1.0	1220	1452.5	1229.97	1450.48

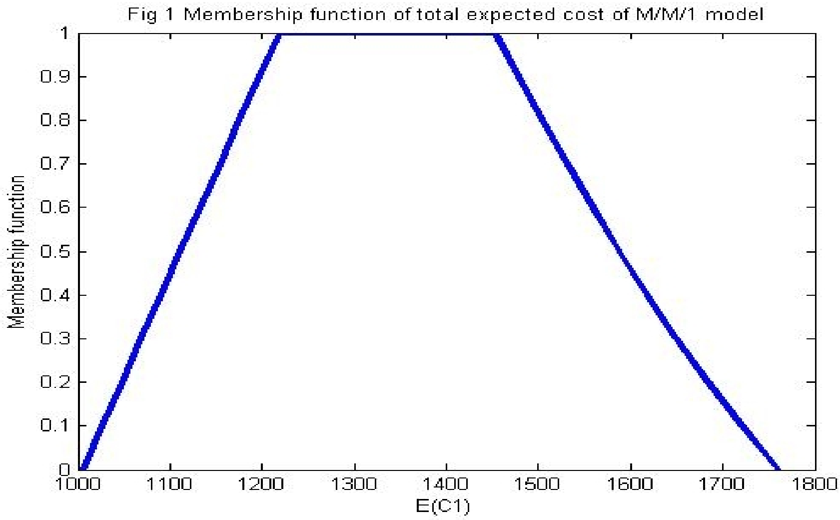

Fig. 1. Membership function of total expected cost of M/M/1 model

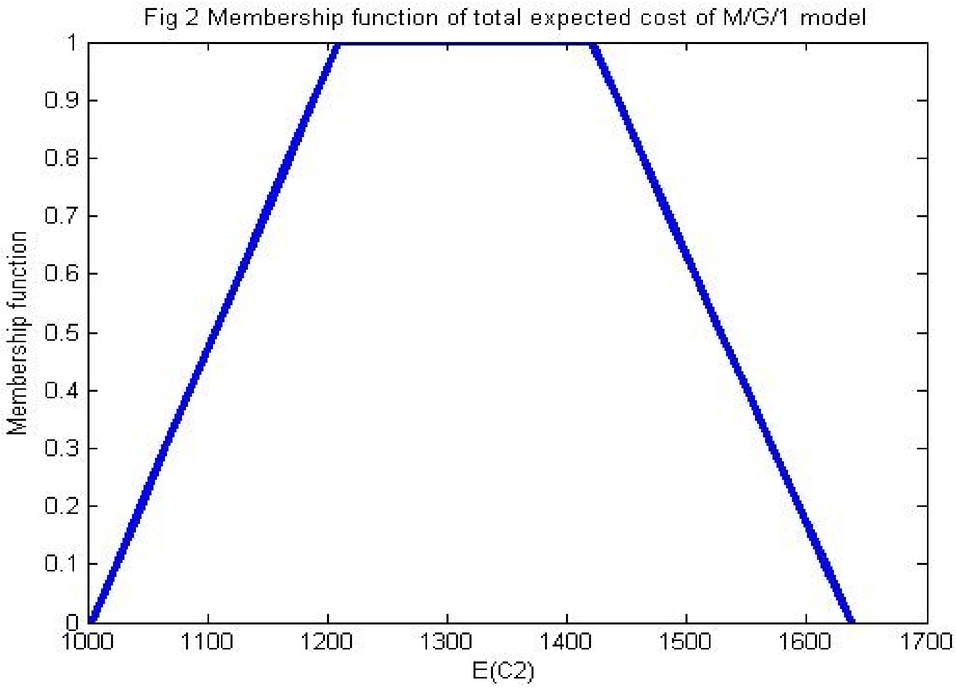

Fig. 2. Membership function of total expected cost of M/G/1 model

With the help of MATLAB 7.04, we perform α-cuts of fuzzy expected total cost of first and second model at eleven distinct α levels 0,0.1,0.2,…..1.0. Crisp intervals for fuzzy expected total cost of first and second model are presented in Table 1. Fig 1 depicts the rough shape of $\eta_{E(\tilde{C}_1)}$.

Fig. 2 depicts the rough shape of $\eta_{E(\tilde{C}_2)}$. The rough shape turns out rather fine and looks like a continuous function. The α-cut represent the probability that these two performance measure will lie in the associated range. Specially, α = 0 the range, the performance measures could appear and for α = 1 the range, the performance measure are likely to be. For example, while these two performance measures are fuzzy, the most likely value of the expected total cost of first model falls between 1220 and 1452.5 , and its value is impossible to fall outside the range of 1006.25 and 1760; it is definitely possible that the expected total cost of second model falls between 1229.97 and 1450.48. approximately, and it will never fall below 1013.69 and above 1677.44.

Now the cost function of first model is E (C_1) = [1006.25, 1220, 1452.5, 1760] and the cost function of second model is E (C_2) = [1013.69, 1229.97, 1450.48, 1677.44].

By comparing the total cost for the both models, it is observed that the result of cost analysis leads to the fuzzy values for costs of each model that has overlapped with each other. To decide about the case which provides the minimum cost, the decision making techniques in uncertain environments have been applied. To compare these costs the fuzzy centroid ranking method is used. By applying the centroid ranking methods it is found that expected total cost of the system after training is minimum, hence it can be concluded that training is useful.

6 Conclusions

The fuzzy queuing model has more applicability in the real environments than the crisp systems. In this paper, two practical systems are compared in real environment. To analyse the conditions more precise and more practical, it is assumed that the rate of arrivals and servicing rate are fuzzy numbers also it is considered that the system costs are fuzzy numbers to express the uncertainty in the system completely. Regarding the conditions of production systems the achieved results can help the decision maker to take the better decisions.

Acknowledgements

The authors are very grateful to the Editor-in-chief, British Journal of Mathematics and Computer Sciences and three anonymous referees for their constructive comments and suggestions, which have been very helpful in improving this paper.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Hillier FS, Lieberman GJ. Introduction to Operations Research, seventh edition, Mc Graw-Hill, Singapore; 2001.

[2] Taha HA. Operations Research: An Introduction, Seventh edition, Prentice Hall, New Jersy; 2003.

[3] Li RJ, Lee ES. Analysis of fuzzy queues, Computers and Mathematics with Applications. 1989;17(7):1143-1147.

[4] Buckley JJ. Elementary queuing theory based on possibility theory, Fuzzy Sets and Systems. 1990;37:43-52.

[5] Buckley JJ, Qu Y. On using α cuts to evaluate fuzzy equations, Fuzzy Sets and Systems 1990;38:309-312.

[6] Negi DS, Lee ES. Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems. 1992;46;321-330.

[7] Chen SP. Parametric nonlinear programming approach to fuzzy queues with bulk service:, European Journal of Operational Research. 2005;163:434-444.

[8] Chen SP. A mathematical programming approach to the machine interference problem with fuzzy parameters, Applied Mathematics and Computation. 2006;174:374-387.

[9] Kao C, Li C, Chen S. Parametric programming to the analysis of fuzzy queues", Fuzzy Sets and Systems. 1999;107:93-100.

[10] Pardo MJ, Fuente D. Optimal selection of the service rate for the finite input source fuzzy queuing system, Fuzzy Sets and Systems. 2006;159:325-342.

[11] Pardo MJ, Fuente D. Optimizing a priority-discipline queuing model using fuzzy set theory, Computers and Mathematics with Applications. 2007;54:267-281.

[12] Barak S, Fallahnezhad MS. Cost analysis of fuzzy queuing systems, International Journal of Applied Operational Research. 2012;2(2):25-36.

[13] Dubois D, Prade H. Operations on fuzzy numbers. International Journal of System Science. 1978;9:613-626.

[14] Ying-Ming Wang, Jian-Bo Yang, Dong-Ling Xu, Kwai-Sang Chin. On the centroids of fuzzy numbers, Fuzzy Sets and Systems. 2006;157:919-926.

[15] Zadeh LA. Fuzzy Sets as a basis for a theory of possibility, Fuzzy Sets and Systems. 1978;1:3-28.

[16] Zimmermann HJ. Fuzzy Set Theory and Its Applications, Fourth Edition, Kluwer-Nijhoff, Boston; 2001.

[17] George J. Klir, Bo Yuan. Fuzzy sets and Fuzzy logic, Theory and applications, PHI learning private Ltd; 2009.

[18] de Cooman. G. A behavioural model for vague probability assessments, Fuzzy Sets and Systems. 2005;154:305-358.

[19] Liou TS, Wang MJJ. Ranking fuzzy numbers with integral value, Fuzzy Sets and Systems, 1992;50;247-255.

[20] Nakumura K. Preference relations on a set of fuzzy utilities as a basis for decision making, Fuzzy Sets and Systems. 1986;20:147-162.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=276&id=6&aid=2262