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An Interactive Model for Fully Fuzzy Multi-level Linear Programming Problem based on Multi-objective Linear Programming Technique

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

An interactive approach is proposed to find the optimal fuzzy solution of fully fuzzy multi-level linear programming (FFMLLP) problem. Firstly, convert the problem under consideration into non-fuzzy multi-level multi-objective linear programming (MLMOLP) problem by using the bound and decomposition method. Secondly, simplify the MLMOLP problem by transforming it into separate multi-objective decision-making problems with hierarchical structure, and solving it by using ε-constraint method. The main results obtained in this paper will be explained by an illustrative numerical example. Finally, compare the proposed approach for solving FFMLLP with The result found in O. Emam et al. [21] to show its effectiveness.

Keywords: Multi-level programming; multi-objective programming; interactive approach; bound and decomposition method; fuzzy linear programming.

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1 Introduction

Fuzzy linear programming (FLP) is the linear programming in which some parameters are fuzzy numbers. The FLP was contrived firstly by Zimmermann [1] who developed a method for solving LP problem using multi-objective functions. Buckley and Feuring [2] presented another method for finding the solution to fuzzy, linear programming problem by changing target function into a linear multi-objective problem.

The FLP in which all decision parameters and variables are fuzzy numbers is called fully fuzzy linear programming (FFLP) Saberi Najafi et al. [3]. Shamooshaki et al. Garg [4] presented an approach for computing the various arithmetic operations using credibility theory corresponding to a different type of intuitionistic fuzzy numbers.By using the concept of the distribution and complementary distribution functions Garg [5], studied the basic arithmetic operations for two generalized positive parabolic fuzzy numbers Ezzati et al. [6] applying new ordering on triangular fuzzy numbers and transforming FFLP to a multi-objective linear programming (MOLP) problem presented a new method to solve FFLP; see also Bhardwaj et al. [7]. Jayalakshmi and Pandian [8] suggested a bound and decomposition method to find an optimal fuzzy solution for the FFLP problem. The introduced method decomposed the FFLP problem into three crisp linear programming with bounded variables constraints and then found the fuzzy optimal solution by solving these problems independently and by using its optimal solutions. Garg [9] studied an approach for solving fuzzy differential equations using Runge-Kutta and Biogeography-based optimization.

Multi-level programming technique is advanced to solve decentralized planning problems with multiple decision makers in a hierarchical organization. Multi-level mathematical programming (MLMP) is identified as mathematical programming that solved decentralized planning problems with multiple executors in a multi-level or hierarchical organization.

The multi-level organization has the following common characteristic:

1. Interactive decision-making units exist within a mostly hierarchical structure.
2. Execution of decision-making is sequential from the top-level to lower-level.
3. Each unit independently maximizes its own net benefits but it affected by the actions of other units.
4. The external effect on a decision maker's problem can be reflected in both the objective function and the set of feasible decision space.

The interactive algorithm uses the concepts of satisfactoriness to multi-objective optimization at each level until a preferred solution is reached. The FLDM gets the satisfactory solutions that are acceptable in rank order to the SLDM. The SLDM will search for the satisfactory solution of the FLDM until the satisfactory solution is reached then the interactive fuzzy methods developed in consideration of fuzziness of human judgment [10,11]. Multi-level multi-objective (MLMOP) programming problems involve sequential or multistage decision making. MLMOP problem is concerned with decentralized planning problems with multiple decision makers have interacted with each other. MLMOP problem is computationally more complex than the conventual's multi-objective programming problem or multi-level programming problem.

In the multi-level field most studies are focused on the bi-level problem [12,13 and 14]. Firstly by finding the convex hull of its original set of constraints then simplifying the equivalent problem by converting it into a separate multi-objective decision-making problem and finally by using the -constraint method the resulted problem is solved.

M.S. Osman et al. [15] proposed a three-planner multi-objective decision-making model and solution method for solving the three-level non-linear multi-objective decision-making (TLNMODM) problem. One may refer to the article ([16,17,18]) for more details on multi-objective.

This paper is organized as follows: Section 2 Fuzzy Concepts. Section 3 formulates the model of fully fuzzy multi-level linear programming problem. Formulation of multi-level multi-objective linear programming problem is obtained in section 4. Subsection 4.1 discusses ε-constraint method. Subsection 4.2 and Subsection 4.3 presents Interactive algorithm for fully fuzzy multi-level multi-objective linear programming problem and flow chart. In addition, a numerical example is provided in Section 5. Finally, conclusion and future works are reported in Section 6.

2 Fuzzy Concepts

In this section we present some of important definitions from the fuzzy set theory, where some of these definitions will be used throughout this thesis and found in [19].

2.1 Fuzzy set

Definition 2.1

A fuzzy set A in R (real line) is defined to be a set of ordered pairs $A=\{(x,\mu_A(x)) | x \in R\}$, where $\mu_A(x)$ is called the membership function for the fuzzy set.

Definition 2.2

A fuzzy set A on R is convex if for any $x, y \in R$ and any $\lambda \in [0,1]$, we have $\mu_A(\lambda x+(1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

Definition 2.3

A fuzzy set A is called normal if there is at least one point $x \in R$ with $\mu_A(x)=1$.

Definition 2.4 [8]

A triangular fuzzy number $\tilde{r}=(r_1,r_2,r_3)$ where $r_1,r_2,r_3 \in R$ and its membership function $\mu_{\tilde{r}}(x)$ is defined as:

$$\mu_{\tilde{r}}(x)=\begin{cases} \frac{x-r_1}{r_2-r_1} & , r_1 \leq x \leq r_2, \\ \frac{x-r_3}{r_2-r_3} & , r_2 \leq x \leq r_3, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Definition 2.5 [8]

Let $\tilde{r}=(r_1,r_2,r_3)$ and $\tilde{s}=(s_1,s_2,s_3)$ are two triangular fuzzy numbers, then the basic arithmetic operations will be defined as follows:

(i) Addition:

$$\tilde{r} \oplus \tilde{s} = \left(r_1 + s_1, r_2 + s_2, r_3 + s_3\right).$$

(ii) Subtraction:

$$\tilde{r} \Theta \tilde{s} = \left(r_1 - s_3, r_2 - s_2, r_3 - s_1\right).$$

(iii) Scalar multiplication:

$$k\tilde{r} = \left(kr_1, kr_2, kr_3\right), \quad \text{if } k \geq 0,$$
$$k\tilde{r} = \left(kr_3, kr_2, kr_1\right), \quad \text{if } k < 0.$$

(iv) Multiplication:

$$\tilde{r} \otimes \tilde{s} = \left(r_1 s_1, r_2 s_2, r_3 s_3\right), r_1 \geq 0,$$
$$\tilde{r} \otimes \tilde{s} = \left(r_1 s_3, r_2 s_2, r_3 s_3\right), r_1 < 0, r_3 \geq 0,$$
$$\tilde{r} \otimes \tilde{s} = \left(r_1 s_3, r_2 s_2, r_3 s_1\right), r_3 < 0.$$

3 Fully Fuzzy Multi-level Linear Programming Problem

Fully fuzzy multi-level linear programming (FFMLLP) problem may be formulated as follows:

$$\begin{aligned}
&\left[1^{st} \text{ level}\right]: \\
&\max_{\tilde{x}_1} \tilde{F}_1 = \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_j, \\
&\text{where } \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n \text{ solves,} \\
&\left[2^{nd} \text{ level}\right]: \\
&\max_{\tilde{x}_2} \tilde{F}_2 = \sum_{j=1}^{n} \tilde{g}_{ij} \otimes \tilde{x}_j, \\
&\text{where } \tilde{x}_3, \ldots, \tilde{x}_n \text{ solves,} \\
&\quad\vdots \\
&\left[m^{th} \text{ level}\right]: \\
&\max_{\tilde{x}_m} \tilde{F}_m = \sum_{j=1}^{n} \tilde{d}_{ij} \otimes \tilde{x}_j, \\
&\text{where } \tilde{x}_{m+1}, \ldots, \tilde{x}_n \text{ solves,}
\end{aligned} \tag{2}$$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{rj} \otimes \tilde{x}_j \leq \tilde{b}_r \ (r = 1, 2, \ldots, k), \tag{3}$$
$$\tilde{x}_j \geq \tilde{0} \ (j = 1, 2, \ldots, n).$$

Where $\tilde{x}_j, (j = 1, 2, \ldots, n)$ be fuzzy variables indicating the i^{th} decision level choice $(i = 1, 2, \ldots, m)$. The parameters $\tilde{c}_{ij}, \tilde{g}_{ij}, \tilde{d}_{ij}, \tilde{a}_{rj}$ and $\tilde{b}_r, (j = 1, 2, \ldots, n)$, $(i = 1, 2, \ldots, m)$, $(r = 1, 2, \ldots, k)$ are fuzzy numbers.

Let the parameters $\tilde{F}_i, \tilde{x}_j, \tilde{c}_{ij}, \tilde{a}_{rj}$ and $\tilde{b}_r$, $(i = 1, 2, \ldots, m), (j = 1, 2, \ldots, n), (r = 1, 2, \ldots, k)$ be the triangular fuzzy numbers (v_{i1}, v_{i2}, v_{i3}), (x_j, y_j, z_j), (a_{rj}, b_{rj}, c_{rj}), (u_{rj}, g_{rj}, e_{rj}) and (p_r, q_r, h_r) respectively. Then the Problem (1)-(2) can be rewritten for i^{th} level in the following form:

$$\left[i^{th}\text{ level}\right]: \max_{(x_i, y_i, z_i)} (v_{i1}, v_{i2}, v_{i3}) = \sum_{j=1}^{n} (a_{rj}, b_{rj}, c_{rj}) \otimes (x_j, y_j, z_j), (i = 1, 2, \ldots, m), \tag{4}$$
$$\text{where } (x_j, y_j, z_j) \text{ solves } (j = i+1, \ldots, n),$$

subject to

$$\sum_{j=1}^{n} (u_{rj}, g_{rj}, e_{rj}) \otimes (x_j, y_j, z_j) \leq (p_r, q_r, h_r), \ (r = 1, 2, ..., k), \tag{5}$$
$$x_j, y_j, z_j \geq 0, (j = 1, 2, \ldots, n).$$

By using the arithmetic operations which obtained in Definition 2.5, then Problem (4)-(5) is decomposed into the following form:

$$\left[i^{th}\text{ level}\right]:$$
$$\max_{x_i} v_{i1} = \sum_{j=1}^{n} \text{lower value of} \left((a_{rj}, b_{rj}, c_{rj}) \otimes (x_j, y_j, z_j) \right), \ (i = 1, 2, \ldots, m),$$
$$\max_{y_i} v_{i2} = \sum_{j=1}^{n} \text{middle value of} \left((a_{rj}, b_{rj}, c_{rj}) \otimes (x_j, y_j, z_j) \right), \ (i = 1, 2, \ldots, m), \tag{6}$$
$$\max_{z_i} v_{i3} = \sum_{j=1}^{n} \text{upper value of} \left((a_{rj}, b_{rj}, c_{rj}) \otimes (x_j, y_j, z_j) \right), \ (i = 1, 2, \ldots, m),$$
$$\text{where } (x_j, y_j, z_j) \text{ solves } (j = i+1, \ldots, n).$$

subject to

$$G = \left\{ \sum_{j=1}^{n} \text{lower value of} \left(\left(u_{rj}, g_{rj}, e_{rj} \right) \otimes \left(x_j, y_j, z_j \right) \right) \le p_r, \ \left(r = 1, 2, \ldots, k \right), \right.$$
$$\sum_{j=1}^{n} \text{middle value of} \left(\left(u_{rj}, g_{rj}, e_{rj} \right) \otimes \left(x_j, y_j, z_j \right) \right) \le q_r, \ \left(r = 1, 2, \ldots, k \right), \tag{7}$$
$$\sum_{j=1}^{n} \text{upper value of} \left(\left(u_{rj}, g_{rj}, e_{rj} \right) \otimes \left(x_j, y_j, z_j \right) \right) \le h_r, \ \left(r = 1, 2, \ldots, k \right),$$
$$\left. x_j, y_j, z_j \ge 0 \ \left(j = 1, 2, \ldots, n \right) \right\}.$$

4 Formulation of Multi-level Multi-objective Linear Programming Problem

By using bound and decomposition method [8], Problem (6)-(7) is converted into multi-level multi-objective linear programming (MLMOLP) problem as follows:

$$\left[1^{st} \text{ level} \right]:$$
$$\max v_{11} = \sum_{j=1}^{n} \text{lower value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right)$$
$$\max v_{12} = \sum_{j=1}^{n} \text{middle value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right) \tag{8}$$
$$\max v_{13} = \sum_{j=1}^{n} \text{upper value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right)$$
$$\text{where} \left(x_2, y_2, z_2 \right), \ldots, \left(x_n, y_n, z_n \right) \text{solves}$$

$$\left[2^{nd} \text{ level} \right]:$$
$$\max v_{21} = \sum_{j=1}^{n} \text{lower value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right)$$
$$\max v_{22} = \sum_{j=1}^{n} \text{middle value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right) \tag{9}$$
$$\max v_{23} = \sum_{j=1}^{n} \text{upper value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \right)$$
$$\text{where} \left(x_3, y_3, z_3 \right), \ldots, \left(x_n, y_n, z_n \right) \text{solves}$$

$$\left[\text{m}^{\text{th}}\ \text{level}\right]:$$
$$\begin{aligned}
&\max v_{m1} = \sum_{j=1}^{n} \text{lower value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right)\\
&\max v_{m2} = \sum_{j=1}^{n} \text{middle value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right)\\
&\max v_{m3} = \sum_{j=1}^{n} \text{upper value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right)\\
&\text{where}\left(x_j, y_j, z_j\right)\text{solves } \left(j = m+1, \ldots, n\right),
\end{aligned} \tag{10}$$

Subject to

$$\begin{aligned}
G = \Big\{ &\sum_{j=1}^{n} \text{lower value of}\left(\left(u_{rj}, g_{rj}, e_{rj}\right) \otimes \left(x_j, y_j, z_j\right)\right) \le p_r,\ \ (r = 1, 2, \ldots, k),\\
&\sum_{j=1}^{n} \text{middle value of}\left(\left(u_{rj}, g_{rj}, e_{rj}\right) \otimes \left(x_j, y_j, z_j\right)\right) \le q_r,\ \ (r = 1, 2, \ldots, k),\\
&\sum_{j=1}^{n} \text{upper value of}\left(\left(u_{rj}, g_{rj}, e_{rj}\right) \otimes \left(x_j, y_j, z_j\right)\right) \le h_r,\ \ (r = 1, 2, \ldots, k),\\
&\qquad\qquad x_j, y_j, z_j \ge 0\ \ \left(j = 1, 2, \ldots, n\right)\Big\}.
\end{aligned} \tag{11}$$

4.1 ε-constraint method [20]

To obtain the preferred solution of the FLDM problem; we transform FLDM problem into the following single objective decision making problem:

$$\left[1^{\text{st}}\ \text{level}\right]:$$
$$\begin{aligned}
&\max v_{11} = \sum_{j=1}^{n} \text{lower value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right)\\
&\ \ \text{Subject to}\\
&\tilde{x} \in G,\\
&\max v_{12} = \sum_{j=1}^{n} \text{middle value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \ge \delta_1,\\
&\max v_{13} = \sum_{j=1}^{n} \text{upper value of}\left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \ge \delta_2.
\end{aligned} \tag{12}$$

Similarly, to obtain the preferred solution of the SLDM problem; we transform problem SLDM into the following single objective decision making problem:

$$\begin{aligned}
&\left[2^{nd}\text{ level}\right]: \\
&\max v_{21} = \sum_{j=1}^{n} \text{lower value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \\
&\quad \text{Subject to} \\
&\tilde{x} \in G, \\
&\max v_{22} = \sum_{j=1}^{n} \text{middle value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \geq \delta_3, \\
&\max v_{23} = \sum_{j=1}^{n} \text{upper value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \geq \delta_4.
\end{aligned} \tag{13}$$

Similarly, to obtain the preferred solution of the m^{th} LDM problem; we transform problem m^{th} LDM into the following single objective decision making problem:

$$\begin{aligned}
&\left[m^{th}\text{ level}\right]: \\
&\max v_{m1} = \sum_{j=1}^{n} \text{lower value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \\
&\text{Subject to} \\
&\tilde{x} \in G, \\
&\max v_{m2} = \sum_{j=1}^{n} \text{middle value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \geq \delta_{2n-1}, \\
&\max v_{m3} = \sum_{j=1}^{n} \text{upper value of} \left((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j)\right) \geq \delta_{2n}.
\end{aligned} \tag{14}$$

Now, we will test whether $(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, ..., \tilde{x}_n^S)$, is preferred solution to the FLDM or it may be changed, by the following test:

If

$$\frac{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^F, \tilde{x}_3^F, ..., \tilde{x}_n^F) - \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, ..., \tilde{x}_n^S) \|_2}{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, ..., \tilde{x}_n^S) \|_2} \prec \delta^F, \tag{15}$$

So $(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, ..., \tilde{x}_n^S)$ is a preferred solution to the FLDM, where δ^F is a small positive constant given by the FLDM which means $(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, ..., \tilde{x}_n^S)$, is a preferred solution of the FFMLMLP problem.

Similarly, we will test whether $(\tilde{x}_1^F, \tilde{x}_2^S, ..., \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}})$, is preferred solution to the $(m-1)^{th}$LDM or it may be changed, by the following test:

If

$$\frac{\| \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, ..., \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{(m-1)^{th}}) - \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, ..., \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}}) \|_2}{\| \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, ..., \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}}) \|_2} \prec \delta^{(m-1)^{th}}, \quad (16)$$

Where $\delta^{(m-1)^{th}}$ is a small positive constant given by the $(m-1)^{th}$LDM which means $(\tilde{x}_1^F, \tilde{x}_2^S, ..., \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}})$, is a preferred solution of the FFMLMLP problem.

4.2 Interactive algorithm for FFMLMLP problem

Step 1: Formulating the FFMLMLP problem go to step 2.

Step 2: Let all fuzzy variables and fuzzy coefficients are triangular fuzzy numbers.

Step 3: Converting the FFMLMLP problem as problem (4) and (5).

Step 4: Set i= 1.

Step 5: Converting the I-LDM problem (4) and (5) into the non-fuzzy model as problem (6) and (7) by using the arithmetic operations on fully fuzzy and by the bound and decomposition method [8].

Step 6: Calculating the individual best and worst solutions for the decomposed problems of the I-LDM problem.

Step 7: If i= n, go to step 8, otherwise, i= i+1, then go to step 5.

Step 8: Set k=0 ; solve the1st level decision-making problem to obtain a set of preferred solutions that are acceptable to the FLDM. The FLDM puts the solutions in order in the format as follows: Preferred solution $(\tilde{x}_1^k, \tilde{x}_2^k, \tilde{x}_3^k), \ldots, (\tilde{x}_1^{k+p}, \ \tilde{x}_2^{k+p}, \tilde{x}_3^{k+p})$

Preferred ranking (satisfactory ranking) $(\tilde{x}_1^k, \tilde{x}_2^k, \tilde{x}_3^k) \succ (\tilde{x}_1^{k+1}, \ \tilde{x}_2^{k+1}, \tilde{x}^{k+1}) \succ \ldots \succ (\tilde{x}_1^{k+p}, \ \tilde{x}_2^{k+p}, \tilde{x}_3^{k+p})$

Step 9: Solving the I-LDM problem by using the ε -constraint method as problem (12), (13) and (14).

Step 10: Given $\tilde{x}_1 = \tilde{x}_1^F$ to the SLDM.

Step 11: If

$$\frac{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^F, \tilde{x}_3^F, \ldots, \tilde{x}_n^F) - \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, \ldots, \tilde{x}_n^S) \|_2}{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, \ldots, \tilde{x}_n^S) \|_2} \prec \delta^F,$$

Where δ^F is a small positive constant given by the FLDM, then go to step 12. Otherwise, go to step 13.

Step 12: $(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S, \ldots, \tilde{x}_n^S)$, is a preferred solution of the FFMLLP problem go to step 14.

Step 13: Set k=k+1, then go to step 8.

Step 14: Similarly, given $\tilde{x}_1 = \tilde{x}_1^F, \tilde{x}_2 = \tilde{x}_2^S, \ldots$ and $\tilde{x}_{n-1} = \tilde{x}_{n-1}^{(m-1)^{th}}$ to the m^{th} LDM

Step 15: If

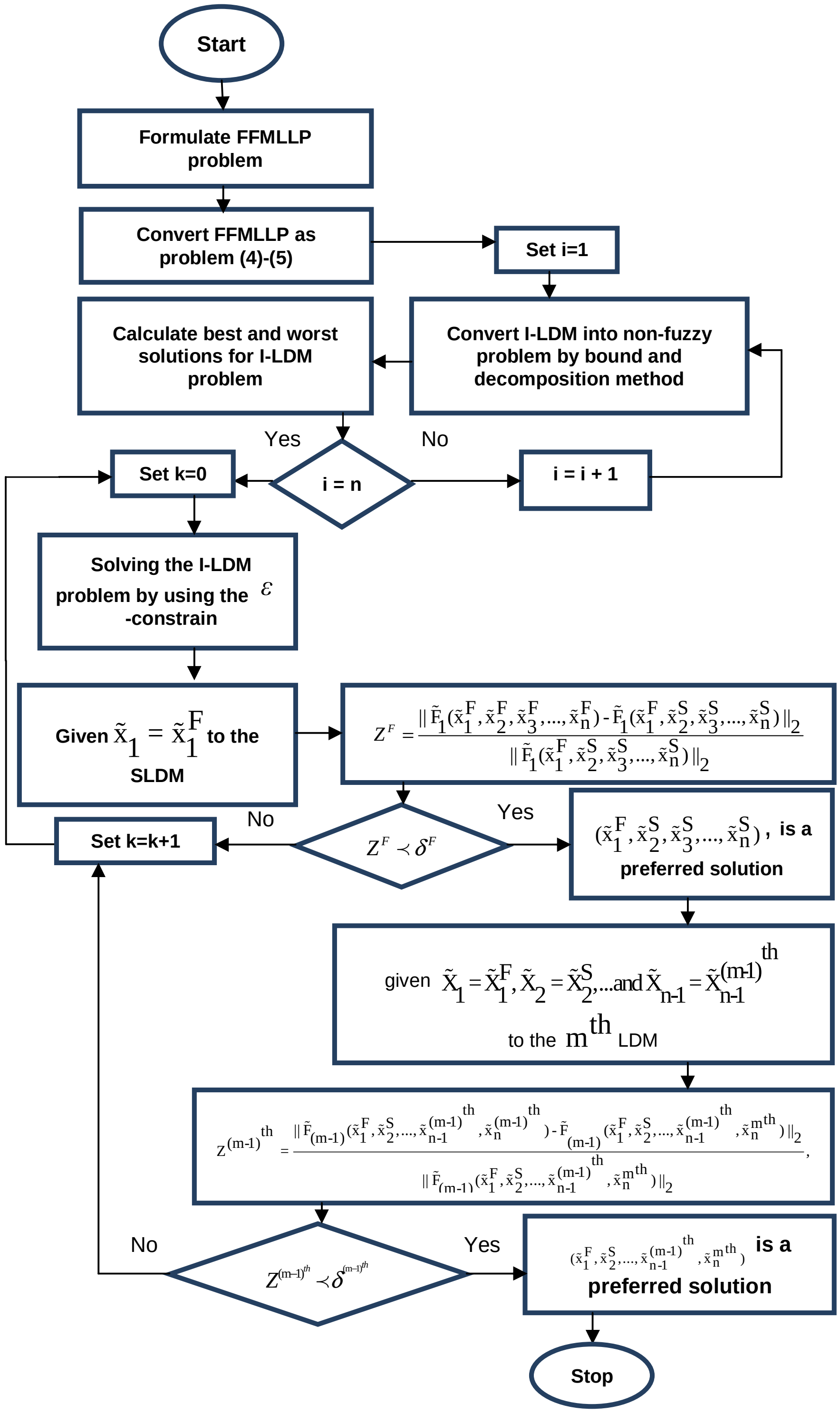
$$\frac{\| \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, \ldots, \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{(m-1)^{th}}) - \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, \ldots, \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}}) \|_2}{\| \tilde{F}_{(m-1)}(\tilde{x}_1^F, \tilde{x}_2^S, \ldots, \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}}) \|_2} \prec \delta^{(m-1)^{th}},$$

Where $\delta^{(m-1)^{th}}$ is a small positive constant given by the $(m-1)^{th}$LDM then go to step 16. Otherwise, go to step 13.

Step 16: $(\tilde{x}_1^F, \tilde{x}_2^S, \ldots, \tilde{x}_{n-1}^{(m-1)^{th}}, \tilde{x}_n^{m^{th}})$ is a preferred solution to the FFMLLP problem go to step 17.

Step 17: Stop.

4.3 A flowchart for solving FFMLLPP



5 Numerical Example

Consider the following example of fully fuzzy three-level linear programming (FFTLLP) problem:

$$
\begin{aligned}
&\left[1^{st}\text{ level}\right]:\\
&\max_{\tilde{x}_1} \tilde{F}_1 = (8,11,15)\otimes\tilde{x}_1 \oplus (1,3,7)\otimes\tilde{x}_2 \oplus (2,5,8)\otimes\tilde{x}_3,\\
&\text{where } \tilde{x}_2, \tilde{x}_3 \text{ solves,}\\
&\left[2^{nd}\text{ level}\right]:\\
&\max_{\tilde{x}_2} \tilde{F}_2 = (4,7,9)\otimes\tilde{x}_1 \oplus (6,10,12)\otimes\tilde{x}_2 \oplus (3,8,11)\otimes\tilde{x}_3,\\
&\text{where } \tilde{x}_3 \text{ solves,}\\
&\left[3^{rd}\text{ level}\right]:\\
&\max_{\tilde{x}_3} \tilde{F}_3 = (7,10,12)\otimes\tilde{x}_1 \oplus (5,9,11)\otimes\tilde{x}_2 \oplus (10,13,16)\otimes\tilde{x}_3,\\
&\text{subject to}\\
&(1,2,3)\otimes\tilde{x}_1 \oplus (5,6,8)\otimes\tilde{x}_2 \oplus (3,5,9)\otimes\tilde{x}_3 \le (20,25,50),\\
&(4,8,11)\otimes\tilde{x}_1 \oplus (1,3,6)\otimes\tilde{x}_2 \oplus (2,3,4)\otimes\tilde{x}_3 \le (18,23,40),\\
&(5,9,10)\otimes\tilde{x}_1 \oplus (2,4,7)\otimes\tilde{x}_2 \oplus (1,2,6)\otimes\tilde{x}_3 \le (27,32,55),\\
&\qquad\qquad \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0.
\end{aligned}
$$

Let

$$\tilde{x}_1 = (x_1, y_1, z_1), \tilde{x}_2 = (x_2, y_2, z_2), \tilde{x}_3 = (x_3, y_3, z_3), \tilde{F}_1 = (v_1, v_2, v_3), \tilde{F}_2 = (v_4, v_5, v_6)$$

and

$\tilde{F}_3 = (v_7, v_8, v_9)$, then the (FFTLLP) problem may be formulated as follows:

$\left[1^{st}\text{ level}\right]$:

$$\max_{(x_1,y_1,z_1)}\left(v_1,v_2,v_3\right)=(8,11,15)\otimes\left(x_1,y_1,z_1\right)\oplus(1,3,7)\otimes\left(x_2,y_2,z_2\right)\oplus(2,5,8)\otimes\left(x_3,y_3,z_3\right),$$

where $\left(x_2,y_2,z_2\right),\left(x_3,y_3,z_3\right)$ solves,

$\left[2^{nd}\text{ level}\right]$:

$$\max_{(x_2,y_2,z_2)}\left(v_4,v_5,v_6\right)=(4,7,9)\otimes\left(x_1,y_1,z_1\right)\oplus(6,10,12)\otimes\left(x_2,y_2,z_2\right)\oplus(3,8,11)\otimes\left(x_3,y_3,z_3\right),$$

where $\left(x_3,y_3,z_3\right)$ solves,

$\left[3^{rd}\text{ level}\right]$:

$$\max_{(x_3,y_3,z_3)}\left(v_7,v_8,v_9\right)=(7,10,12)\otimes\left(x_1,y_1,z_1\right)\oplus(5,9,11)\otimes\left(x_2,y_2,z_2\right)\oplus(10,13,16)\otimes\left(x_3,y_3,z_3\right),$$

subject to

$$(1,2,3)\otimes\left(x_1,y_1,z_1\right)\oplus(5,6,8)\otimes\left(x_2,y_2,z_2\right)\oplus(3,5,9)\otimes\left(x_3,y_3,z_3\right)\le(20,25,50),$$

$$(4,8,11)\otimes\left(x_1,y_1,z_1\right)\oplus(1,3,6)\otimes\left(x_2,y_2,z_2\right)\oplus(2,3,4)\otimes\left(x_3,y_3,z_3\right)\le(18,23,40),$$

$$(5,9,10)\otimes\left(x_1,y_1,z_1\right)\oplus(2,4,7)\otimes\left(x_2,y_2,z_2\right)\oplus(1,2,6)\otimes\left(x_3,y_3,z_3\right)\le(27,32,55),$$

$$x_1,x_2,x_3,y_1,y_2,y_3,z_1,z_2,z_3\ge 0.$$

By using the arithmetic operations in Definition 2.5 and the bound and decomposition method [8], therefore the (FFTLLP) problem above became non-fuzzy three- level multi-objective linear programming (TLMOLP) problem as follows:

$\left[1^{st}\text{ level}\right]$:

$$\max_{x_1} v_1 = 8x_1 + x_2 + 2x_3,$$

$$\max_{y_1} v_2 = 11y_1 + 3y_2 + 5y_3,$$

$$\max_{z_1} v_3 = 15z_1 + 7z_2 + 8z_3,$$

where $\left(x_2,y_2,z_2\right),\left(x_3,y_3,z_3\right)$ solves,

$\left[2^{nd}\text{ level}\right]$:

$\max_{x_2} v_4 = 4x_1 + 6x_2 + 3x_3,$

$\max_{y_2} v_5 = 7y_1 + 10y_2 + 8y_3,$

$\max_{z_2} v_6 = 9z_1 + 12z_2 + 11z_3,$

where $\left(x_3, y_3, z_3\right)$ solves,

$\left[3^{rd}\text{ level}\right]$:

$\max_{x_3} v_7 = 7x_1 + 5x_2 + 10x_3,$

$\max_{y_3} v_8 = 10y_1 + 9y_2 + 13y_3,$

$\max_{z_3} v_9 = 12z_1 + 11z_2 + 16z_3,$

subject to

$$
\begin{aligned}
G = \{&x_1 + 5x_2 + 3x_3 \le 20,\\
&4x_1 + x_2 + 2x_3 \le 18,\\
&5x_1 + 2x_2 + x_3 \le 27,\\
&2y_1 + 6y_2 + 5y_3 \le 25,\\
&8y_1 + 3y_2 + 3y_3 \le 23,\\
&9y_1 + 4y_2 + 2y_3 \le 32,\\
&3z_1 + 8z_2 + 9z_3 \le 50,\\
&11z_1 + 6z_2 + 4z_3 \le 40,\\
&10z_1 + 7z_2 + 6z_3 \le 55,\\
&x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \ge 0\}.
\end{aligned}
$$

After applying the bound and decomposition method on the (TLMOLP) problem, the individual best and worst fuzzy solutions are obtained in the following tables:

Table 1. The individual best fuzzy solution of the TLMOLP problem

	Individual best fuzzy solution		
	FLDM (i=1)	**SLDM (i=2)**	**TLDM (i=3)**
$\tilde{x}_1^*$	$(1.176,\ 1.176,\ 1.839)$	$(1.5,\ 1.5,\ 1.5)$	$(1.176,\ 1.765,\ 1.839)$
$\tilde{x}_2^*$	$(1.047,\ 0,\ 0)$	$(3.667,\ 3.667,\ 3.667)$	$(0,\ 0,\ 0)$
$\tilde{x}_3^*$	$(4.529,\ 4.529,\ 4.943)$	$(0,\ 0,\ 0.375)$	$(4.529,\ 4.529,\ 4.943)$
$\tilde{F}_i^*$	$(19.518,\ 35.588,\ 67.126)$	$(28.002,\ 47.167,\ 61.625)$	$(53.522,\ 70.647,\ 101.149)$

Table 2. The individual worst fuzzy solution of the TLMOLP problem

	Individual worst fuzzy solution		
	FLDM (i=1)	**SLDM (i=2)**	**TLDM (i=3)**
$\tilde{x}_1^-$	$(0,\ 0,\ 1.176)$	$(0,\ 0,\ 1.5)$	$(0,\ 0,\ 1.176)$
$\tilde{x}_2^-$	$(0,\ 0,\ 0)$	$(0,\ 0,\ 3.667)$	$(0,\ 0,\ 0)$
$\tilde{x}_3^-$	$(0,\ 0,\ 4.529)$	$(0,\ 0,\ 0)$	$(0,\ 0,\ 4.529)$
$\tilde{F}_i^-$	$(0,\ 0,\ 53.882)$	$(0,\ 0,\ 57.5)$	$(0,\ 0,\ 86.588)$

Using ε-constraint method [20] and the solution of FLDM problem, its equivalent single objective function can be formulated as follows:

$\left[1^{st}\ \text{level}\right]$:

$$\max_{x_1} v_1 = 8x_1 + x_2 + 2x_3,$$

subject to

$$\tilde{x} \in G,$$
$$11y_1 + 3y_2 + 5y_3 \geq 35.588,$$
$$15z_1 + 7z_2 + 8z_3 \geq 67.126.$$

Where $\delta_{12} = (b_{12} - a_{12})s_1 + a_{12} = 35.588$ and $\delta_{13} = (b_{13} - a_{13})s_1 + a_{13} = 67.126$.

Then the solution of FLDM is $x_1 = 4.5, x_2 = 0, x_3 = 0, y_1 = 1.176, y_2 = 0,$ $y_3 = 4.529, z_1 = 1.839,\ z_2 = 0,\ z_3 = 4.943$, $v_1 = 36$ where $s_1 = 1$ and $\delta^F = 0.5$ are given by FLDM.

Using ε-constraint method [20] and the solution of SLDM problem, its equivalent single objective function can be formulated as follows:

$\left[2^{nd}\ \text{level}\right]$:

$$\max_{x_2} v_4 = 4x_1 + 6x_2 + 3x_3,$$

subject to

$$\tilde{x} \in G,$$
$$7y_1 + 10y_2 + 8y_3 \geq 42.45,$$
$$9z_1 + 12z_2 + 11z_3 \geq 61.21.$$

Where $\delta_{22} = \left(b_{22} - a_{22}\right)s_2 + a_{22} = 42.45$ and $\delta_{23} = \left(b_{23} - a_{23}\right)s_2 + a_{23} = 61.21.$

So the solution of SLDM is

$$x_1 = 3.68, x_2 = 3.26, x_3 = 0, y_1 = 0.214, y_2 = 4.095, y_3 = 0, z_1 = 0, z_2 = 5.1, z_3 = 0,$$
$$v_4 = 34.3158 \text{ and } s_2 = 0.9.$$

By using the FLDM test function to decide whether the solution is acceptable or not so $\left(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S\right)$ is preferred solution to the FLDM upon following test:

$$\frac{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^F, \tilde{x}_3^F) - \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S) \|_2}{\| \tilde{F}_1(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S) \|_2} \le \delta^F,$$

Where $\tilde{x}_1^F = (4.5, 1.177, 1.839), \tilde{x}_2^S = (3.26, 4.095, 5.1), \tilde{x}_3^S = (0,0,0),$

Then $\dfrac{\left\|(36, 35.592, 67.121) - (21.26, 25.232, 63.285)\right\|_2}{\left\|(21.26, 25.232, 63.285)\right\|_2} = 0.258 \prec 0.5$

So $(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S)$ is a preferred solution of the FFMLLP problem.

Similarly, do the same way on the SLDM and TLDM to have the following results:

By applying ε-constraint method [20] and the solution of the TLDM, its equivalent single objective function can be formulated as follows:

$$\begin{aligned}
&\left[3^{rd} \text{ level}\right]: \\
&\max_{x_3} v_7 = 7x_1 + 5x_2 + 10x_3 \\
&\text{subject to} \\
&\tilde{x} \in G, \\
&10y_1 + 9y_2 + 13y_3 \ge 70.647, \\
&12z_1 + 11z_2 + 16z_3 \ge 101.149.
\end{aligned}$$

Where $\delta_{32} = \left(b_{32} - a_{32}\right)s_3 + a_{32} = 70.647$ and $\delta_{33} = \left(b_{33} - a_{33}\right)s_3 + a_{33} = 101.149.$

The solution of TLDM is

$$x_1 = 1.4, x_2 = 0, x_3 = 6.2, y_1 = 1.176, y_2 = 0, y_3 = 4.529, z_1 = 1.839, z_2 = 0,$$

$z_3 = 4.943, v_7 = 71.8$ where $s_3 = 1$ and $\delta^S = 0.5$ are given by SLDM.

By using the SLDM test function to decide whether the solution is acceptable or not so $\left(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^T\right)$ is preferred solution to the SLDM upon following test:

$$\frac{\| \tilde{F}_2(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^S) - \tilde{F}_2(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^T) \|_2}{\| \tilde{F}_2(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^T) \|_2} \le \delta^S,$$

Where $\tilde{x}_1^F = (4.5, 1.177, 1.839), \tilde{x}_2^S = (3.26, 4.095, 5.1)$ and $\tilde{x}_3^T = (6.2, 4.529, 4.943)$

Then $\dfrac{\left\|(37.56, 53.34, 77.751) - (56.16, 89.572, 132.124)\right\|_2}{\left\|(56.16, 89.572, 132.124)\right\|_2} = 0.40147 \prec 0.5$

So $\left(\tilde{x}_1^F, \tilde{x}_2^S, \tilde{x}_3^T\right) = \left((4.5, 1.177, 1.839), (3.26, 4.095, 5.1), (6.2, 4.529, 4.943)\right)$ is a preferred solution of the FFMLLP problem, which means that the optimal fuzzy solution is

Table 3. The individual best fuzzy solution of the TLMOLP problem

	Individual best fuzzy solution		
	FLDM (i=1)	**SLDM (i=2)**	**TLDM (i=3)**
$\tilde{x}_1^*$	$(4.5,\ 1.176,\ 1.839)$	$(3.68,\ 0.214,\ 0)$	$(1.4,\ 1.176,\ 1.839)$
$\tilde{x}_2^*$	$(0,\ 0,\ 0)$	$(3.26,\ 4.095,\ 5.1)$	$(0,\ 0,\ 0)$
$\tilde{x}_3^*$	$(0,\ 4.529,\ 4.943)$	$(0,\ 0,\ 0)$	$(6.2,\ 4.529,\ 4.943)$
$\tilde{F}_i^*$	$(36,\ 35.581,\ 67.129)$	$(34.28,\ 42.448,\ 61.2)$	$(71.8,\ 70.637,\ 101.156)$

Finally, by using the result found in O. Emam et al. [21] we get a better result in the proposed algorithm other one we can introduce this at this table

In comparing between the result found in O. Emam et al. [21] and the proposed algorithm, the result shows that the proposed algorithm better than the result found in O. Emam et al. [21], the table below introduce the following:

Table 4. Comparison between the result found in O. Emam et al. [21] and the proposed algorithm

Level	**The result found in O. Emam et al. [21]**	**The proposed algorithm**
FLDM	(12.536,31.54,58.059)	(36,35.581,67.129)
SLDM	(16.344,45.814,70.25)	(34.28,42.448,61.2)
TLDM	(18.526,59.326,84.489)	(71.8,70.637,101.156)

6 Conclusion

This paper was proposed an interactive approach to find the solution of fully fuzzy multi-level linear programming problem where all of its decision parameters and variables are fuzzy numbers. Firstly, the problem under consideration was converted into non-fuzzy multi-level multi-objective linear programming (MLMOLP) problem by using the bound and decomposition method. Secondly, the MLMOLP problem be simplified by transforming it into separate multi-objective decision-making problems with hierarchical structure, and solved it by using ε-constraint method.

However, there are many other aspects, which should be reconnoitered and studied in the area of fuzzy multi-level optimization such as:

1. Interactive fully fuzzy multi-level multi-objective integer linear fractional programming problem.
2. Interactive fully fuzzy multi-level multi-objective integer linear quadratic programming problem.
3. Interactive fully fuzzy fully rough multi-level multi-objective integer programming problem.

Competing Interests

Authors have declared that no competing interests exist.

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