



Practical Realization of Simultaneous Equations Solutions with Elimination Method and Its Effects on Students Performance in Some Selected Schools in Ekiti and Ondo States of Nigeria

O. O. Fagbohun^{1*} and O. A. Oni²

¹Department of Electrical Engineering, Faculty of Engineering, Ekiti State University, Ado-Ekiti, Nigeria.

²Department of Civil Engineering, Faculty of Engineering, Ekiti State University, Ado-Ekiti, Nigeria.

Authors' contributions

This work was carried out in collaboration between both authors. Author OOF designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author OAO managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

Introducing students into Simultaneous equations and Matrices problem solving techniques without a practical way of transforming a real life physical system into the theory of what makes the subject, makes it so abstract and uninteresting, with many students avoiding its attempt in examinations. In order to redress the problem associated with abstract nature, a simultaneous equations and matrices training kits consisting of a simultaneous stepped-faced base, with a sizeable number of multi-colored cuboid inserted-numbered modeled cube, counters stand, counting number plates and arithmetic signs standing plates was developed for the practical teaching of Simultaneous equations in Schools. The two distinct differently colored cubes forms characters x and y to represent physical systems been used to represent values which of course are distinct from one another. 8 Secondary Schools in Ekiti and Ondo States of the Federal

*Corresponding author: Email: olumuyiwafagbohun@gmail.com, engrginspirationsi@yahoo.com;

Republic of Nigeria with a class range of 82 to 124, with same teacher administering the subject over years, were selected for evaluation of the impact of the developed device in giving a clearer picture of the principles of solving simultaneous equations and matrices in order to strengthen students knowledge of applying the subject to varied physical problems. The percentage response of students that attempted, accurately solved the problems with and without the use of the kits were determined with respect to time of contacts. The results from the sampled Schools in use of the kits, shows that the total number of students attempt to questions on the Simultaneous equations in the whole class range between 80% and 91%, with between 82% and 97% correct solutions obtained, with between 3 and 4 contact class teachings of 45 minutes per class in comparison to the earlier 4 to 5 class teachings without the use of practical kits with between 22% and 40% of the total population of the school class well grounded in the methods of its solutions. Thus, the practical training kits improves the perception of the theory of Simultaneous equations, and strengthen the students in interpreting the underline principles.

Keywords: Simultaneous equations; attempted; accuracy; impact; performance; practical teaching.

1. INTRODUCTION

In mathematics, a set of simultaneous equation also known as a system of equation or an equation system, is a finite set of equations for which common solutions are sought [1,2]. An equation system is usually classified in the same manner as single equations, namely as a: i). System of linear equations, ii). system of non linear equations, iii). System of bilinear equations, iv). System of polynomial equations, v). System of ordinary differential equations vi). System of partial differential equations, vii). System of partial differential equations or a system of difference equations [2,3,4].

A system of linear equations is a collection of two or more linear equations involving the same set of variable [1,3], for example

$$4x + 2y + z = 7 \quad (1)$$

$$2x - 2y + 3z = 3 \quad (2)$$

$$-x + \frac{1}{2}y - \frac{1}{2}z = -1 \quad (3)$$

is called a system of three equations of equations 1,2 and 3 in the three variable x, y, z. A solution to a linear system is an assignment of values to the variable x, y, z such that all the equations are simultaneously satisfied, and in which to the system of equations 1,2 and 3 gives $x = y = z = 1$.

The theory of linear systems is the basic and a fundamental part of linear algebra, a subject which is used in most part of modern mathematics [2,3]. Computational algorithms for finding solutions are an important part of numerical linear algebra, and play a prominent

role in Engineering, physics, computer science and even economics [2,4,5]. A system of non-linear equations can often be approximated by a linear system [3,4,5,6], a helpful technique when making a mathematical model or computer simulation of a relatively complex system [4,5,7].

The practical method of introducing the subject of simultaneous equations problem solving for linear systems at the basic level schools [8,9,10,11], for a better understanding of this important part of linear algebra is always a concern to teachers, with many students in examinations, avoiding its selection to attempt.

This research work explain the use of a practical kit developed to demystify the abstract nature of introduction of simultaneous equations solving to students, in a friendly-colored class environment, by translating a given physical model, with other means of information and data carrier into a mathematical form, giving rise to a simultaneous equations problem using elimination method, and the effect of its use in 2 states of the Federal republic of Nigeria.



Fig. 1. Simultaneous equations and Matrices Modules Stand

2. MATERIALS AND METHODS

The Simultaneous and Matrices equations kit [12] consists of (a) Two module stand 500 mm x 230 mm x 125 mm (b) 75 mm x75 mm cubes consisting of 12 green and 12 blue cuboid imprints with numbers (c) 2 plus (+) and minus (-) and equal to (=) stands (d) 45 plate numbered kits sticks. In all simultaneous equations, the characters X, Y, Z are been used to represent values which of course are distinct from one another [8,9,10,11]. The teacher is expected to first explain to his students that the character x, y, z in the equation $x + y + z = 0$ represents a distinct values of either positive or negative numbers, which may equally be an integer or a fraction. In the practical work to be developed using this kit, the teacher must explain that:

- (a) A green cube with its imprinted numbers can be used to represent a character x of a distinct value to be found or g etc, and
- (b) A blue cube with its imprinted numbers can be used to represent a character y or b of a distinct value to be found.

For example; with the use of the kit, we can solve a simple simultaneous equation of equations 4 and 5 given below [8,9,10,11,13]

$$2x + y = 4 \quad (4)$$

$$x + 2y = 5 \quad (5)$$

by following this steps in practical form [12], as exemplified in Fig. 2.



Fig. 2. Set-up of the example given in practical form

In practical sense x is not y and y is not z and, in this case, Color presentations are used, so the three colours are different, and represents different information that needs to be stressed.

After the instructor / teacher introduction of the characters required [8,9,10,11,12], we choose green cube to represent character x. Also a Blue cube can be used to represent character y or b etc (i.e. color blue carries value y) and a yellow cube to represent a character z or any other suitable character within an equation. It is to be noted that for the idea to be permanent in the mind of the student, the instructor needs to explain that;

- (1) A green color is not the same as a blue color and,
- (2) That the value of the green cube may be or not equal to that of a blue cube due to color differences i.e. $x < y$ or $x > y$ or $x = y$.

To solve the simultaneous equation of equations 4 and 5 above with the kit, place the simultaneous equation stand of Fig. 1 already assembled on a flat surface, and explain that the stand has two flat surfaces to be used. The top surface of the simultaneous board is to carry equation 4 and the flat horizontal surface carry equation 5 as shown in Fig. 2.

To the class, let the green cube represents the character x and the blue cubes to represent the character y (or vice -versa). Set the equation given as equation 4 on the top surface of the simultaneous board as shown in Fig. 2 and equation 5 on the lower surface of the simultaneous board. This is accomplished by: (a) taking 2 green cubes with one inserted cuboids on the top of the simultaneous board facing the class, or 1 green cube with 2 inserted cuboids, to represent $2x$, and same principle for the blue cubes to represent $2y$. To eliminate one of the characters, multiply equation 4 by 2 to give $4x + 2y = 8$. At this junction, the equation 4 plane (on top of the simultaneous board) will change by setting $4x + 2y = 8$ on the surface which should be done with students participation. The lower part (i.e. equation 5) is equally multiplied by 1, to give $x + 2y = 5$.

Then looking at the upper and lower part of the simultaneous board surface together we have

$$4x + 2y = 8 \quad (6)$$

$$x + 2y = 5 \quad (7)$$

Watching the 2 equations of equations 6 and 7, we note that the blue cubes of equation 6 and 7 are same, and subtracting the two from each other gives zero. Thus, if we subtract equation 7

on the board from equation 6 on the top board we arrive practically at $(4x-x) + (2y-2y) = 8 - 5$ which on the board looks (4 green cubes - 1 green cubes) + (2 blue cubes - 2 blue cubes) = $8 - 5$, which gives 3 green cube = 3. Thus we say $x = 1$, since 1 green cube represents x .

The set up is as shown in Figs. 3 and 4.



Fig. 3. Set-up of the example given using elimination method in practical form



Fig. 4. Another way of Setting-up of Fig. 2 example given in practical form

When we solve as explained for y we get $y=2$.

To reinforce the students knowledge with the practical kits in the solving of Simultaneous equations using elimination method, the 2nd example is administered as given in equations 8 and 9.

Again, as a second example, we can set up the following simple equation g , in which case, the earlier steps explained is repeated, i.e. with the equation 8 and 9

$$x + y = 4 \quad (8)$$

$$2x - 3y = 3 \quad (9)$$

- (a) Explain that the simultaneous equations stands has 2 flat surfaces (steps). The top

surface is to carry equation 8 and the flat horizontal surface carry equation 9 as illustrated in Fig. 5.

- (b) Let the Green cubes represents the character X and Blue cubes to represent the character Y (or vice - versa).
 (c) Set the equation given with the cubes on the stand as shown in Fig. 5.



Fig. 5. Set-up of the example given in practical form

- (d) To eliminate one of the characters. Multiply equation 8 by 2 to give $2x + 2y = 8$ and set it out on the stand with students participation, as shown in Fig. 6.



Fig. 6. Solving the problem using elimination method



Fig. 7. The left over after subtraction

- (e) Subtract equation 9 from equation 8 i.e (2 green cuboids -2 green Cuboid) = 0 taking note of sign change where applicable, solve for the character that is left as shown in Fig. 7.

i.e $5y = 5 \implies y = 1$

Representing $5y = 5$ or $y = 1$

- (f) Go back to the initial equation 8 or 9, substitute the value of y , and solve for x , where we found $x = 3$.
- (g) This is the practical solution using elimination principle of solving a simultaneous equation.



Fig. 8. Some modules of Simultaneous and Matrices kits

The teacher still have more time in minutes to solve more equations for explanation in practical form (i.e.) As given below; as problem 1,

$$2x - y = 1 \tag{10}$$

$$3x + 2y = 5 \tag{11}$$

with answers to problem of equations 10 and 11 arrived at as : $x = 1, y = 1$. The practical settings of the simultaneous equations is as shown in Fig. 9, in front of the class.



Fig. 9. Set up of the example given in problem 1

For problem 2, we have,

$$x + 2y = 8 \tag{12}$$

$$4x - y = 9 \tag{13}$$

with answers to problem of equations 12 and 13 arrived at as: $x = 3, y = 3$. The practical set up of this problem is as shown in Fig. 10, and the steps enumerated above is easily followed in the practical solving of the problem using elimination method.



Fig. 10. Set up of the example given in problem 2

With the practical introduction in solving 3 or more problems with the use of elimination method, in the first 45 minutes of class introduction to the simultaneous equations methods of solving, the substitution method of solving the problems is now introduced as well as simple form of converting simultaneous equations to matrices as exemplified below, and methods of its solutions.

The simultaneous equations can be used to introduce students to matrices for example, the simultaneous equation of equations 14 and 15 given below in the third or fourth class meetings to deepen the knowledge of the use of simultaneous equations.

$$x + y = 4 \tag{14}$$

$$2x - 3y = 3 \tag{15}$$

which can be expressed in matrix form as shown in equation 16.

$$\begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{16}$$

The teacher is expected to explain the cross multiplication and sign changes involved in this method of conversion and follow the explanations in the solving of simultaneous equations in practical form.

3. RESULTS AND DISCUSSION ON EFFECTS ON STUDENTS PERFORMANCE

In a sampled 8 secondary schools with the use of the practical kits in teaching students Simultaneous Equations and Matrices, with a class range of 82 to 124 students at ss3 classes per school, with the same teacher administering the subject over the years, the following results were obtained. From the sampled school result on students response to questions on Simultaneous Equations and Matrices principles and applications in year 2011, it was discovered as in Table 1 that between 46% and 75% of the total students population in their final year class, senior secondary school, attempted to pick the problems on Simultaneous Equations for solving, in which case, between 40% and 56% of those students that tried to solve the problems arrived at a required accurate solution. It was also discovered that between 23% and 41% of the total population of the schools at the senior level were well grounded in the methods of solutions of Simultaneous Equations problems. This observations were also verified for another set of students in the same school , in their final year senior secondary levels in year 2012 and 2013, in which case, for year 2012, it was discovered as in Table 2 that between 27% and 65% of the total students population in their final year class, senior secondary school, attempted to pick the problems on Simultaneous Equations for solving , in which case, between 41% and 68% of those students that tried to solve the problems arrived at a required accurate solution. It was also discovered that between 11% and 38% of the total population of the schools at the senior level were well grounded in the methods of solutions of Simultaneous Equations problems. For year 2013, it was also discovered that between 27% and 66% of the total students population in their final year class, senior secondary school, attempted to pick the problems on Simultaneous Equations for solving , in which case, between 48% and 60% of those students that tried to solve the problems arrived at a required accurate solution. It was also discovered that between 18% and 39% of the total population of the schools at the senior level were well grounded in the methods of solutions of Simultaneous Equations problems.

However, in year 2014, set theory practical training kits as detailed above were administered in the teaching of the new set of final year senior

secondary school students, and the following results were obtained. Between 89% and 98% of the total students population in their final year class, senior secondary school, attempted to pick the problems on Simultaneous Equations for solving, in which case, between 82% and 96% of those students that tried to solve the problems arrived at a required accurate solution. It was also discovered that between 80% and 91% of the total population of the schools at the senior level were well grounded in the methods of solutions of Simultaneous Equations problems. To confirm the impact of the application of set practical training kits in the study of the subject to students, another data were obtained for the year 2015, in which case, it was discovered that Between 82% and 98% of the total students population in their final year class, senior secondary school, attempted to pick the problems on Simultaneous Equations for solving, in which case, between 90% and 97% of those students that tried to solve the problems arrived at a required accurate solution. It was also discovered that between 80% and 91% of the total population of the schools at the senior level were well grounded in the methods of solutions of Simultaneous Equations problems.

On the average, From Table 1, we deduce that out of 635 total number of students in their final year in 8 different secondary Schools in year 2011, an average of 59.26% of the total number of students sampled attempted to solve questions on simultaneous equations, with 48.71% of the students attempting the questions, arriving at an accurate solution to the problems, and an average of 28.65% of the total class of 635 students obtaining an accurate solution to the problems. Table 2 shows the results obtained for the year 2012 with the same sampled 8 schools with 715 number of students in their final year senior secondary level classes, in which case, an average of 46.71% of the total number of students sampled attempted to solve questions on simultaneous equations, with 51.67% of the students attempting the questions, arriving at an accurate solution to the problems, and an average of 22.79% of the total class of 715 students obtaining an accurate solution to the problems. In year 2013, Table 3 shows the results obtained with the same sampled 8 schools with 635 number of students in their final year senior secondary level classes, in which case, an average of 50.27% of the total number of students sampled attempted to solve questions on simultaneous equations, with

Table 1. Year 2011 sampled schools report on students response to questions on simultaneous equations and matrices before the use of practical kits

	School A	School B	School C	School D	School E	School F	School G	School H
Total number of students in class	102	94	76	95	81	62	53	72
Total number 45mins/Class teachings	5	4	5	4	4	4	5	5
Number of students that attempted Simultaneous equations questions	74	56	45	43	45	42	34	36
Number of students that got correct answers to questions	41	27	24	22	19	17	15	19
Percentage of students that attempted questions on Simultaneous equations (%)	72.55	59.57	59.21	45.26	55.56	67.74	64.15	50
Percentage of students that accurately solved problems on Simultaneous equations (%)	55.41	48.21	53.33	51.16	42.22	40.47	44.11	52.77
Overall Percentage of students that accurately solved problems on Simultaneous equations out of total class (%)	40.20	28.72	31.57	23.15	23.45	27.41	28.30	26.38

Table 2. Year 2012 Sampled Schools report on students response to questions on Simultaneous Equations and Matrices before the use of practical kits

	School A	School B	School C	School D	School E	School F	School G	School H
Total number of students in class	91	108	89	76	99	87	72	93
Total number 45mins/Class teachings	5	4	5	4	4	4	5	5
Number of students that attempted Simultaneous equations questions	59	51	56	39	27	24	31	46
Number of students that got correct answers to questions	34	31	27	19	18	10	15	19
Percentage of students that attempted questions on Simultaneous equations (%)	64.83	47.22	62.92	51.31	27.27	27.58	43.05	49.46
Percentage of students that accurately solved problems on Simultaneous equations (%)	57.62	60.78	48.21	48.71	66.67	41.67	48.38	41.30
Overall Percentage of students that accurately solved problems on Simultaneous equations out of total class (%)	37.36	28.70	30.33	25	18.18	11.49	20.83	20.43

Table 3. Year 2013 Sampled Schools report on students response to questions on Simultaneous Equations and Matrices before the use of practical kits

	School A	School B	School C	School D	School E	School F	School G	School H
Total number of students in class	108	91	81	98	90	72	68	76
Total number 45mins/Class teachings	5	4	5	4	4	4	5	5
Number of students that attempted Simultaneous equations questions	71	41	40	37	42	37	38	46
Number of students that got correct answers to questions	42	24	22	18	23	21	20	24
Percentage of students that attempted questions on Simultaneous equations (%)	65.74	45.05	49.38	27.55	46.67	51.38	55.88	60.52
Overall Percentage of students that accurately solved problems on Simultaneous equations (%)	59.15	58.53	35	48.64	54.76	56.75	52.63	52.17
Percentage of students that accurately solved problems on Simultaneous equations out of total class (%)	38.88	26.37	27.16	18.36	25.55	29.16	29.41	31.57

Table 4. Year 2014 Sampled Schools report on students response to questions on Simultaneous Equations and Matrices with the use of practical kits

	School A	School B	School C	School D	School E	School F	School G	School H
Total number of students in class	114	101	92	99	74	81	76	89
Total number 45mins/Class teachings	3	3	3	4	3	3	4	4
Number of students that attempted Simultaneous equations questions	111	92	86	89	72	76	74	83
Number of students that got correct answers to questions	92	88	79	82	67	71	67	79
Percentage of students that attempted questions on Simultaneous equations (%)	97.36	91.08	93.47	89.89	97.29	93.82	97.36	93.25
Percentage of students that accurately solved problems on Simultaneous equations (%)	82.88	95.65	91.86	92.13	93.05	93.42	90.54	95.18
Overall Percentage of students that accurately solved problems on Simultaneous equations out of total class (%)	80.70	87.12	85.86	82.82	90.54	87.65	88.13	88.76

Table 5. Year 2015 Sampled Schools report on students response to questions on Simultaneous Equations and Matrices with the use of practical kits

	School A	School B	School C	School D	School E	School F	School G	School H
Total number of students in class	101	113	79	87	85	106	78	84
Total number 45mins/Class teachings	3	3	3	4	3	3	4	4
Number of students that attempted Simultaneous equations questions	94	110	72	86	83	95	64	72
Number of students that got correct answers to questions	91	106	66	75	78	86	60	68
Percentage of students that attempted questions on Simultaneous equations (%)	93.06	97.34	91.13	98.85	97.64	89.62	82.05	87.71
Percentage of students that accurately solved problems on Simultaneous equations (%)	96.80	96.36	91.67	87.20	93.97	90.52	93.75	94.44
Percentage of students that accurately solved problems on Simultaneous equations out of total class (%)	90.09	84.31	83.54	86.20	91.76	81.13	76.92	80.95

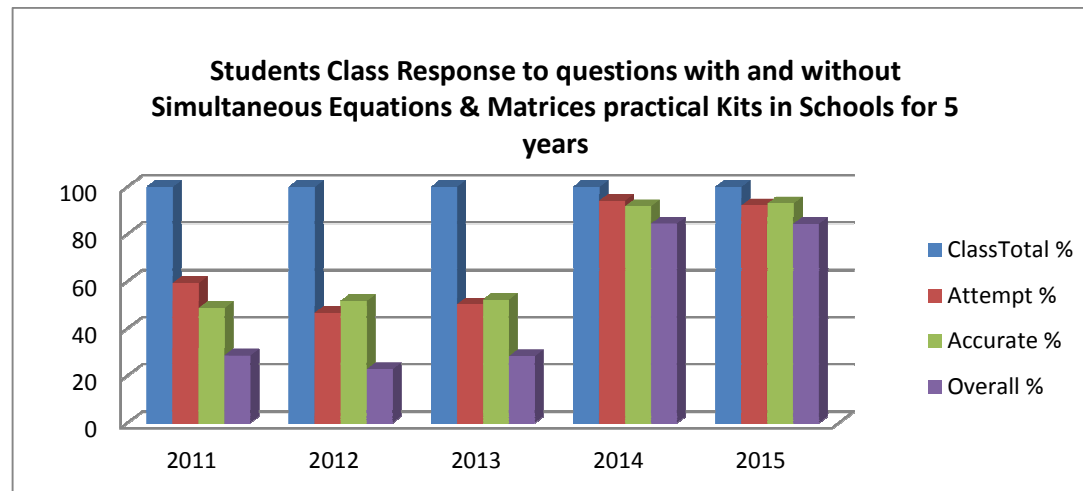


Fig. 11. Class Percentage of response to questions on Simultaneous Equations and Matrices in 5 years

52.20% of the students attempting the questions, arriving at an accurate solution to the problems, and an average of 28.31% of the total class of 635 students obtaining an accurate solution to the problems. For year 2014, with the use of simultaneous training kits as discussed in the teaching of the students as a practical way of introduction to the subject, out of a total class of 726 students in the sampled schools.

Table 4 shows the results with an average of 94.19% of the total number of students sampled attempted to solve questions on simultaneous equations, with 91.84% of the students attempting the questions, arriving at an accurate solution to the problems, and an average of 86.45% of the total class of 726 students obtaining an accurate solution to the problems. The same trend of result was validated in year 2015, with 733 students in the sampled schools, in which case, as derived from Table 5, an average of 92.18% of the total number of students sampled attempted to solve questions on simultaneous equations, with 93.09% of the students attempting the questions, arriving at an accurate solution to the problems, and an average of 84.36% of the total class of 733 students obtaining an accurate solution to the problems. An histogram chart of the results obtained between the year 2011 to 2015, on the selected schools for sampling is as shown in Fig. 11. The graph of Fig. 11 shows that in year 2014 and 2015, there is a great improvement in class response to teachers teaching in Simultaneous and Matrices Theory, leading to a high attempt of questions and accurate solutions obtained. Thus, it is of note to say, that the application of practical kits in the teaching of Simultaneous Equations to students at the upper senior secondary level, had a great impact on the students assimilation of the subject matter, and necessary for new generation classes for easier understanding.

4. CONCLUSION

Mathematical analysis of concepts is very important in the study of Engineering without which, a prospective Engineer is more or less an historian. Practical realization of Simultaneous and Matrices theory will assist more in driving home the necessary concepts required of modern Engineers, and the application of the use of practical Simultaneous equations and Matrices kits in the teaching of Simultaneous equations to students in Schools, had proved from the assessment results conducted in 8

different Schools with a wide spread in different local government areas of Ekiti and Ondo State of Nigeria, with considerable class population to have a great impact on the understanding and response level of students to its methods of approach in problem solving, than without; in which case, a good percentage of between 80% and 91% total students responded to all its practicing questions, with between 82% and 97% correct solutions obtained. The total class contact meetings was equally reduced from between 4 and 5 contacts, to 3 and 4 contact meetings. It is therefore recommended that practical kits in Simultaneous equations should be employed in all Senior Secondary Schools in the teaching of the subject matter, to impact more knowledge of its use in daily affairs in the training of Students.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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