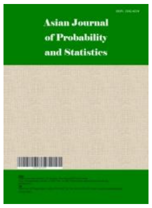


Asian Journal of Probability and Statistics

14(1): 1-11, 2021; Article no.AJPAS.66824
ISSN: 2582-0230

Samade Probability Distribution: Its Properties and Application to Real Lifetime Data

Samuel Aderoju[1*]

[1]Department of Statistics and Mathematical Sciences, Kwara State University, Malete, P.M.B. 1530, Ilorin, Kwara State, Nigeria.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v14i130317
Editor(s):
(1) Dr. Manuel Alberto M. Ferreira, Lisbon University, Portugal.
Reviewers:
(1) Jesús Alfredo Fajardo González, University of Orient. Cumaná, Republic of Venezuela.
(2) Pedro Luiz Ramos, University of São Paulo, Brazil.
Complete Peer review History: https://www.sdiarticle4.com/review-history/66824

Original Research Article

Received: 17 January 2021
Accepted: 23 March 2021
Published: 19 July 2021

Abstract

A new two-parameter lifetime distribution has been proposed in this study. The distribution is called Samade distribution. The model is motivated by the wide use of the lifetime models derived from the mixture of gamma and exponential distributions. Its mathematical properties which include the first four moments, variance as well as coefficient of variation, reliability function, hazard function, survival function, Renyi entropy measure and distribution of order statistics have been successfully derived. The maximum likelihood estimation of its parameters and application to real life data have been discussed. Application of this model to three real datasets shown that the proposed model yields a satisfactorily better fit than other existing lifetime distributions. The comparism of goodness-of-fits were established using -2Loglikelihood, AIC and BIC.

Keywords: Samade distribution; moments; reliability measure; probability of order statistics; maximum likelihood estimation.

Mathematics subject classification: *60E05, 62E15.*

**Corresponding author: E-mail: samuel.aderoju@kwasu.edu.ng;*

1 Introduction

In the recent time, many authors have proposed different distributions for modeling real lifetime data, which are paramount in many applied sciences like biomedical, environmental, insurance and finance, engineering, to mention a few. These distributions, such as generalized exponential and gamma, are quite flexible in nature and are appropriate for modeling a wide range of data showing constant, monotonic or non-monotonic failure rate property [1].
Recently, a one-parameter distribution called Pranav distribution was proposed by Shukla [2]. The distribution is a mixture of exponential distribution with scale parameter θ and gamma distribution with shape parameter 4 and scale parameter θ while their mixing proportions are $\frac{\theta^4}{\theta^4+6}$ and $\frac{6}{\theta^4+6}$.

$$f_p(x|,\theta) = pg_1(x;\theta) + (1-p)g_2(x;4,\theta),$$

where

$$p = \frac{\theta^4}{\theta^4+6}, \quad g_1(x;\theta) = \theta e^{-\theta x} \quad and \quad g_2(x;4,\theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}$$

Therefore, the probability density function (pdf) of the Pranav distribution, $f_p(x|\theta)$, was defined as:

$$f_p(x|,\theta) = \frac{\theta^4}{\theta^4+6}(\theta + x^3)e^{-\theta x}, \qquad x > 0$$

where

$$\theta > 0.$$

Several other work have been done on mixture and generalization of Pranav distribution by many authors including Uwaeme and Akpan [3], Umeh and Ibenegbu [4]. The Sujatha distribution was proposed by Shanker [5]. The author studied the properties of the model and compared its goodness of fit with some existing models to model biomedical sciences data. Similarly, much work have been done on the generalization and mixture of Sujatha distribution by different researchers including Shanker and Hagos [6], Aderoju [7], Tesfay and Shanker [8].

Lindley [9] used a mixture of exponential and length-biased exponential distributions to illustrate the difference between fiducial and posterior distributions [10]. This mixture is called the Lindley distribution (LD). More studies have been done on the properties, generalization and mixture of the Lindley distribution by Ghitany et al. [11], Nadarajah et al. [12], Ghitany et al. [13], Shanker and Mishra [14], Zakerzadeh and Dolati [15], Aderoju et al. [16], Oluyede and Yang [10].

The aim of this paper is to propose, study the properties and application of a new two-parameter distribution called Samade distribution, which is a mixture of exponential and gamma distributions at a specific proportion.

The proposed probability distribution is presented in Section 2 and study its mathematical properties in section 3. The corresponding Maximum Likelihood estimates are discussed in Section 4. In Section 5, we discuss its application to real life examples and we present concluding remarks in Section 6.

2 Materials and Methods

In this section, the probability density function (pdf) of a new exponential-gamma mixture distribution called Samade distribution (SD) is derived, which is given by

$$f(x|\alpha,\theta) = \begin{cases} \frac{\theta^4}{\theta^4+6\alpha}(\theta + \alpha x^3)e^{-\theta x}, & if\ x > 0 \\ 0, & elsewhere \end{cases}, \qquad (1)$$

where $\theta > 0$ is a scale parameter and $\alpha > 0$ is a shape parameter. The two-parameter lifetime distribution is a mixture of exponential (θ) and gamma (4, θ) distributions, we have

$$f(x|\alpha,\theta) = pg_1(x;\theta) + (1-p)g_2(x;4,\theta),$$

where

$$p = \frac{\theta^4}{\theta^4 + 6\alpha}, \quad g_1(x;\theta) = \theta e^{-\theta x} \quad and \quad g_2(x;4,\theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}$$

Hence, (1) is defined as:

$$f(x|\alpha,\theta) = \frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^3)e^{-\theta x}, \qquad x > 0$$

where

$\theta > 0$ and $\alpha > 0$.

Note that when $\alpha = 1$ and $\alpha = 0$, $f(x|\alpha,\theta)$ reduces to the one-parameter Pranav distribution and exponential distribution, respectively.

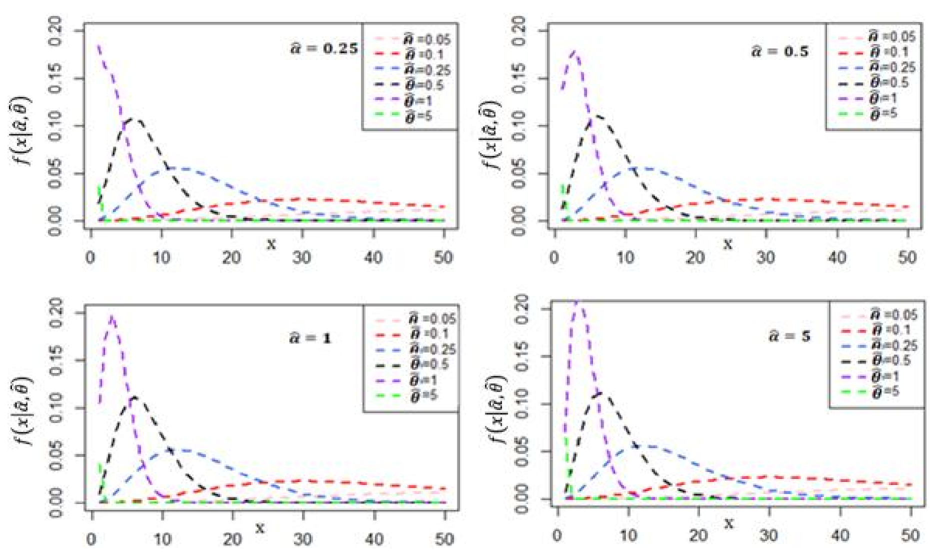
Moreover, the corresponding cumulative distribution function (CDF) of (1) is:

$$F(x) = 1 - \frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4}e^{-x\theta}, \quad x > 0 \qquad (2)$$

where

$\theta > 0$ and $\alpha > 0$.

Graphs of (1) and (2) are shown in Figs. 1 and 2 for varying values of parameters θ and α respectively.



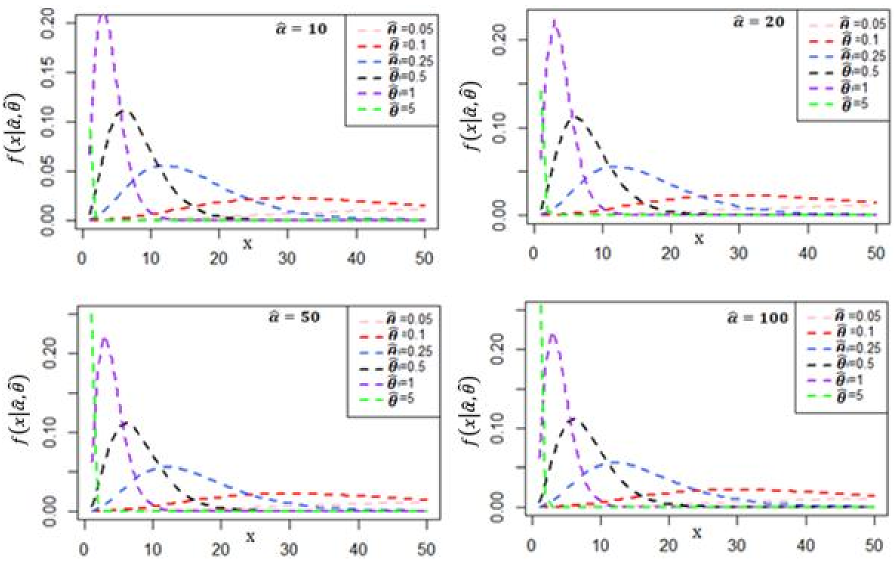


Fig. 1. Pdf plots of Samade distribution of varying values of parameters, $\hat{\theta}$ and $\hat{\alpha} = 10, 20, 50, 100$

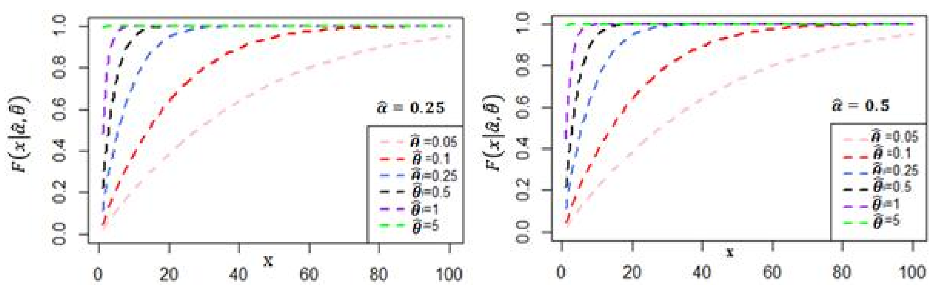


Fig. 2. CDF plots of Samade distribution of varying values of parameters, $\hat{\theta}$ and $\hat{\alpha} = 0.25, 0.5$

3 Mathematical Properties

We present some of the mathematical characteristics of the Samade distribution in this section. The properties include the moments, reliability analysis, entropy and the order statistics.

3.1 Moments of the Samade distribution

The r^{th} moment of a random variable X with Samade distribution is given by

$$E(X^r) = \mu_r = \int_0^\infty x^r f(x|\alpha,\theta)dx,$$

where r is a positive integer.

$$= \int_0^\infty x^r \frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^3)e^{-\theta x}dx = \frac{r!\,\theta^4 + (r+3)!\,\alpha}{\theta^r(\theta^4 + 6\alpha)} \tag{3}$$

From (3), we can express the first four moment as $(r = 1, 2, 3, 4)$:

$$\mu_1 = \frac{24\alpha + \theta^4}{6\alpha\theta + \theta^5}$$

$$\mu_2 = \frac{2(60\alpha + \theta^4)}{6\alpha\theta^2 + \theta^6}$$

$$\mu_3 = \frac{6(120\alpha + \theta^4)}{6\alpha\theta^3 + \theta^7}$$

$$\mu_4 = \frac{840}{\theta^4} - \frac{816}{6\alpha + \theta^4}$$

Note that, the variance (σ^2) of the random variable X can be obtained as:

$$\sigma^2 = E(X^2) - [E(X^1)]^2 = \mu_2 - [\mu_1]^2 = \frac{144\alpha^2 + 84\alpha\theta^4 + \theta^8}{(6\alpha\theta + \theta^5)^2}$$

The coefficient of variation (CV) and the index of dispersion (γ) of SD are obtained as:

$$CV = \frac{\sigma}{\mu_1} = \frac{\sqrt{\frac{144\alpha^2+84\alpha\theta^4+\theta^8}{(6\alpha\theta+\theta^5)^2}}}{\frac{24\alpha+\theta^4}{6\alpha\theta+\theta^5}} = \frac{\sqrt{(144\alpha^2 + 84\alpha\theta^4 + \theta^8)}}{(24\alpha + \theta^4)(6\alpha\theta + \theta^5)}$$

$$\gamma = \frac{\sigma^2}{\mu_1} = \frac{144\alpha^2 + 84\alpha\theta^4 + \theta^8}{144\alpha^2\theta + 30\alpha\theta^5 + \theta^9}$$

3.2 Renyi's entropy

The Renyi's entropy of a random variable *X* with density function, $f(x|\alpha,\theta)$, as defined by Renyi [17] is given by

$$R_H(x|\alpha,\theta) = \frac{1}{1-p} log \int_0^\infty [f(x|\alpha,\theta)]^p dx \tag{4}$$

$$= \frac{1}{1-p} log \int_0^\infty \left[\frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^3)e^{-\theta x}\right]^p dx$$

$$= \frac{1}{1-p} \log\left[\int_0^\infty \frac{\theta^{4p}}{(\theta^4 + 6\alpha)^p}(\theta + \alpha x^3)^p e^{-\theta p x} dx\right]$$

$$= \frac{1}{1-p} \log\left[\int_0^\infty \frac{\theta^{4p}}{(\theta^4 + 6\alpha)^p}\left(1 + \frac{\alpha}{\theta}x^3\right)^p e^{-\theta p x} dx\right]$$

$$= \frac{1}{1-p} \log\left[\frac{\theta^{4p}}{(\theta^4 + 6\alpha)^p}\sum_{j=0}^{\infty}\binom{p}{j}\left(\frac{\alpha}{\theta}\right)^j \int_0^\infty x^{3j+1-1}e^{-\theta p x} dx\right]$$

$$= \frac{1}{1-p} \log\left[\frac{\theta^{4p}}{(\theta^4 + 6\alpha)^p}\sum_{j=0}^{\infty}\binom{p}{j}\left(\frac{\alpha}{\theta}\right)^j \frac{\Gamma(3j+1)}{(\theta p)^{3j+1}}\right]$$

where

$p \geq 0$ with $p \neq 0$.

3.3 Order statistics

Order statistics gives one of the popular fundamental tools for obtaining inference related to reliability data. For independent copies $X_1,\ X_2,\ \ldots, X_n$ of a random variable X with Samade distribution, we have that the largest and smallest orders are denoted and defined by $X_n = \max(X_1,\ X_2,\ \ldots, X_n)$ and $X_1 = \min(X_1,\ X_2,\ \ldots, X_n)$ respectively. Here, n is the sample size. Observe that $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ are order observations of $X_1,\ X_2,\ \ldots, X_n$. Moreover, the pdf of $X_{(k)}$, k^{th} order statistic can be expressed as

$$f_{X(k)}(x|\alpha,\theta) = \frac{n!}{(k-1)!\,(n-k)!} f(x)[F(x)]^{k-1}[1-F(x)]^{n-k} \tag{5}$$

From (5) we can write

$$f_{X(k)}(x|\alpha,\theta) = \frac{n!\,\theta^4}{(k-1)!\,(n-k)!\,(\theta^4+6\alpha)}(\theta + \alpha x^3)e^{-\theta x}\left[1 - \frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4}e^{-x\theta}\right]^{k-1}\left[\frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4}e^{-x\theta}\right]^{n-k}$$

where

$p \geq 0$ with $p \neq 0$.

The functions of the first and n^{th} order statistics are:

$$f_{X(1)}(x|\alpha,\theta) = \frac{n\theta^4}{(n-1)!\,(\theta^4+6\alpha)}(\theta + \alpha x^3)e^{-\theta x}\left[\frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4}e^{-x\theta}\right]^{n-1}$$

and

$$f_{X(n)}(x|\alpha,\theta) = \frac{n\theta^4}{(\theta^4+6\alpha)}(\theta + \alpha x^3)e^{-\theta x}\left[1 - \frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4}e^{-x\theta}\right]^{n-1}$$

Respectively, where $p \geq 0$ with $p \neq 0$.

3.4 Reliability analysis

The reliability analysis of any given pdf is always considered based on the survival function, S, and the hazard rate function, H, which were derived as shown below.

3.4.1 Survival function

The survival function is generally defined as the probability that an item does not fail prior to some time. It is expressed as

$$S(x|\alpha,\theta) = 1 - F(x|\alpha,\theta) = \frac{\left(\theta^4 + \alpha(6 + 6x\theta + 3x^2\theta^2 + x^3\theta^3)\right)}{6\alpha + \theta^4} e^{-x\theta},$$

3.4.2 Hazard rate function

The hazard rate function can be expressed as the conditional probability of failure, given that it has survived to the time. It is given by

$$h(x|\alpha,\theta) = \frac{f(x|\alpha,\theta)}{S(x|\alpha,\theta)} = \frac{\frac{\theta^4}{\theta^4+6\alpha}(\theta + \alpha x^3)e^{-\theta x}}{\frac{(\theta^4+\alpha(6+6x\theta+3x^2\theta^2+x^3\theta^3))}{6\alpha+\theta^4} e^{-x\theta}} = \frac{\theta^4(x^3\alpha + \theta)}{\theta^4 + \alpha(6 + 6x\theta + 3x^2\theta^2 + x^3\theta^3)}$$

where

$\theta > 0$ and $\alpha > 0$.

Figs. 3 and 4 represent the graph of the survival function and hazard rate function of the Samade distribution for varying values of the parameters α and θ. The hazard rate function graph shows a monotonically increasing shape.

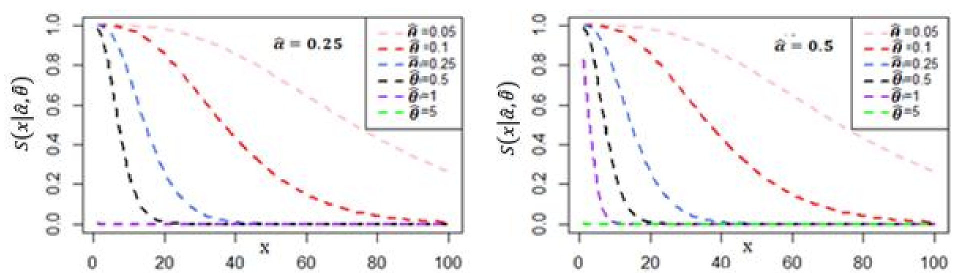


Fig. 3. Survival function plots of Samade distribution for varying values of the parameters $\widehat{\theta}$ and $\widehat{\alpha}$

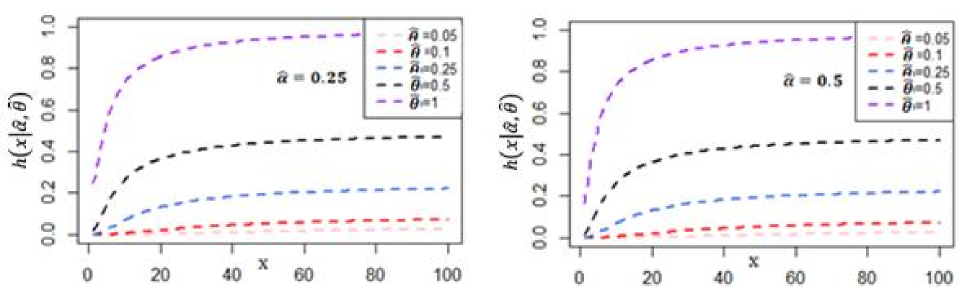


Fig. 4. Hazard rate function plot of Samade distribution for varying values of the parameters $\widehat{\theta}$ and $\widehat{\alpha}$

4 Maximum Likelihood Estimation

Suppose $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size *n* from the Samade distribution, the maximum likelihood function of parameters can be written as

$$L(\alpha,\theta)=\prod_{i=1}^{n}\frac{\theta^4}{\theta^4+6\alpha}(\theta+\alpha {x_i}^3)e^{-\theta x_i},$$

and the log-likelihood function is

$$\ell(\alpha,\theta)=\log\left(L(\alpha,\theta)\right)=nlog(\theta)-n4log(\theta)-nlog(6\alpha)+\sum_{i=1}^{n}\log(\theta+\alpha {x_i}^3)-\theta\sum_{i=1}^{n}x_i$$

Differentiating the $\log\left(L(\alpha,\theta)\right)$ partially with respect to associated parameters we have

$$\frac{\partial \log\left(L(\alpha,\theta)\right)}{\partial\alpha}=-\frac{n}{\alpha}+\sum_{i=1}^{n}{x_i}^3(\theta+\alpha {x_i}^3)^{-1}=0 \tag{6}$$

$$\frac{\partial \log\left(L(\alpha,\theta)\right)}{\partial\theta}=\frac{n}{\theta}-\frac{4n}{\theta}+\sum_{i=1}^{n}(\theta+\alpha {x_i}^3)^{-1}-\sum_{i=1}^{n}x_i=0 \tag{7}$$

where

$\theta>0$ and $\alpha>0$.

The Maximum Likelihood Estimates (MLEs), $\hat{\alpha}\ and\ \hat{\theta}$, of α and θ are solutions of equation (6) and (7). Obviously, analytical expressions for $\hat{\alpha}\ and\ \hat{\theta}$ are not available. Hence, we computed the MLEs numerically using the *nloptr* package and *bobyqa* function in R software [18].

5 Application to Real Dataset

In this section we used three datasets as detailed below to evaluate the application of the Samade distribution (SD) and compared its goodness of fit with Exponential distribution (ED), quasi Lindly distribution (QLD) by Shanker and Mishra [14], quasi-Sujatha distribution (QSD) by Shanker [19] and Pranav distribution (PD). To evaluate and compare the goodness of fit of the distribution with the newly introduced SD, we obtain the MLEs' of the parameters, -2lnL, Akaike Information Criteria (AIC) and Bayesian information criteria (BIC).

Dataset 1: This dataset is the strength data of glass of the aircraft window reported by Fuller et al. [20].

18.83 20.80 21.657 23.03 23.23 24.05 24.321 25.5 25.52 25.8 26.69
26.77 26.78 27.05 27.67 29.9 31.11 33.20 33.73 33.76 33.89 34.76
35.75 35.91 36.98 37.08 37.09 39.58 44.045 45.29 45.381

Dataset 2: The following dataset represents the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20 mm taken from Bader and Priest [21].

1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585

Dataset 3: The dataset represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England; they are taken from Smith and Naylor [22].

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.84, 1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.70, 1.78, 1.89

Table 1. MLEs', -2lnL, AIC and BIC of the fitted models of the three datasets

Dataset	Model	Expected Mean (Observed mean)	MLEs	-2lnL	AIC	BIC
Data 1 n=31	SD	30.8114 (30.8114)	$\hat{\alpha} = 40.1552$ $\hat{\theta} = 0.1298$	232.7728	236.7727	239.6407
	PD	30.8117 (30.8114)	$\hat{\theta} = 0.1298$	232.7752	234.7752	239.6432
	QSD	30.8086 (30.8114)	$\hat{\alpha} = 0.0050$ $\hat{\theta} = 0.0959$	241.3166	245.3165	248.1845
	QLD	30.7873 (30.8114)	$\hat{\alpha} = 0.0.003$ $\hat{\theta} = 0.0649$	252.3182	256.3182	259.1862
	ED	30.8017 (30.8114)	$\hat{\theta} = 0.0325$	274.5288	276.5289	281.3969
Data 2 n=69	SD	2.4514 (2.4513)	$\hat{\alpha} = 9998.9$ $\hat{\theta} = 1.6316$	166.6939	170.6939	175.1621
	PD	2.5965 (2.4513)	$\hat{\theta} = 1.2251$	217.1224	219.1224	225.5906
	QSD	2.4511 (2.4513)	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 1.0806$	198.4925	202.4925	206.9607
	QLD	2.4507 (2.4513)	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.8157$	211.4644	215.4644	219.9326
	ED	2.4513 (2.4513)	$\hat{\theta} = 0.4079$	261.7352	263.7352	270.2034
Data 3 n=63	SD	1.5071 (1.5068)	$\hat{\alpha} = 7499.1$ $\hat{\theta} = 2.6518$	93.7870	97.7870	102.0733
	PD	1.6072 (1.5068)	$\hat{\theta} = 1.5607$	180.9627	182.9627	189.249
	QSD	1.5067 (1.5068)	$\hat{\alpha} = 0.0003$ $\hat{\theta} = 1.6872$	124.3834	128.3834	132.6696
	QLD	1.5067 (1.5068)	$\hat{\alpha} = 0.0.003$ $\hat{\theta} = 1.3272$	132.6521	136.6521	140.9384
	ED	1.5068 (1.5068)	$\hat{\theta} = 0.6636$	177.6606	179.6606	185.9469

In Table 1 above, we present the observed mean with the corresponding expected mean, values of MLEs' of the parameter of the models, -2lnL, AIC and BIC. We find that the proposed Samade distribution fit the three datasets better than the Pranav distribution, QSD, QLD and Exponential distribution.

6 Concluding Remarks

The Samade distribution, which is a mixture of exponential and gamma distributions, has been successfully developed; its mathematical properties which include the first four moments, variance, CV, reliability function, hazard function, survival function, quantile, Renyi entropy measure and distribution of order statistics have been successfully derived. The plots of its pdf, CDF, hazard function and survival function are presented at varying values of the parameters. Application of this model were made to three real life datasets and the results shown that the proposed model gives a satisfactorily better fit consistently than exponential, Pranav, quasi-Lindley and quasi-Sujatha distributions for all the three datasets.

Acknowledgement

The author is very grateful to the Editorial team and the two reviewers for their helpful and constructive comments, which improved the manuscript greatly.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Maurya1 RK, Tripathi1 YM, Lodhi1 C, Rastogi MK. On a generalized Lomax distribution; Int. J. Syst Assur Eng Manag; 2019.
Available:https://doi.org/10.1007/s13198-019-00839-0

[2] Shukla KK. Pranav distribution with properties and its applications. Biom Biostat Int J. 2018;7(3):244–254.
DOI: 10.15406/bbij.2018.07.00215

[3] Uwaeme O, Akpan NP. An extended pranav distribution. Asian Journal of Probability and Statistics. 2018,2019;2(4):1-15.
Article no.AJPAS.46556

[4] Umeh E, Ibenegbu A. A Two-parameter pranav distribution with properties and its application. J Biostat Epidemiol. 2019;5(1):74-90.

[5] Shanker R. Sujatha distribution and its applications. Statistics in Transition New Series. 2016a;17(3):391-410.

[6] Shanker R, Hagos F. On Poisson-Sujatha distribution and its applications to Biological Sciences. Biometrics and Biostatistics International Journal. 2016;2(4):1-5.

[7] Aderoju SA. A new generalized poisson-sujatha distribution and its applications. Applied Mathematical Sciences. 2020;14(5):229–234.
Available:https://doi.org/10.12988/ams.2020.914185

[8] Tesfay M, Shanker R. Another Two-Parameter Sujatha Distribution with Properties and Applications. Journal of Mathematical Sciences and Modelling. 2019;2(1):1-13.
DOI: http://dx.doi.org/10.33187/jmsm.416628

[9] Lindley DV. Fiducial distributions and Bayes' theorem. J R Statist Soc. 1958;20:102–107.

[10] Oluyede BO, Yang T. A new class of generalized Lindley distributions with applications. Journal of Statistical Computation and Simulation; 2014.
Available:http://dx.doi.org/10.1080/00949655.2014.917308

[11] Ghitany ME, Atieh B, Nadarajah S. Lindley distribution and its applications. Math Comput Simulation. 2008;73:493–506.

[12] Nadarajah S, Bakouch SH, Tahmasbi R. A generalized Lindley distribution. Sankhya B. 2011;73:331–359.

[13] Ghitany ME, Al-Mutairi DK, Balakrishnan N, Al-Enezi LJ. Power Lindley distribution and associated inference. Comput Statist Data Anal. 2013;64:20–33.

[14] Shanker R, Mishra A. 2013 A quasi Lindley distribution. Afr J Math Comput Sci. 2013;6(4):64–71.

[15] Zakerzadeh H, Dolati A. Generalized lindley distribution. J Math Extension. 2009;3:13–25.

[16] Aderoju SA, Jolayemi ET, Ibrahim AO. A Comparative Study on Zero-truncated Generalized Poisson-Lindley and Zero-truncated Poisson-Lindley Distributions, International Journal of Mathematical Archive. 2017;8(3):92-98.

[17] Renyi A. On measures of entropy and information, in proceedings of the 4th berkeley symposium on Mathematical Statistics and Probability, Berkeley University of California Press. 1961;1:547-561.

[18] R Core Team. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria; 2020.
Available:https://www.R-project.org/

[19] Shanker R. A quasi Sujatha distribution, Internat. J. Probab. Stat. 2016b;5(4):89–100.

[20] Fuller EJ, Frieman S, Quinn J. Fracture mechanics approach to the design of glass aircraft windows: A case study. SPIE Proc. 1994;(2286):419- 430.

[21] Bader MG, Priest AM. Statistical aspects of fier and bundle strength in hybrid composites, In; Hayashi T, Kawata K, Umekawa S, editors. Progress in Science in Engineering Composites, ICCM-IV, Tokyo. 1982;1129-1136.

[22] Smith RL, Naylor JC. A comparison of Maximum likelihood and Bayesian estimators for the three parameter Weibull distribution. Applied Statistics. 1987;36:358-369.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle4.com/review-history/66824