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(φ, ψ)-Contractıon Condition for Multivalued Mappings in Complete Modular Metric Spaces

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper we investigated (φ, ψ)-contractıon condition for multivalued type mappings in complete modular metric spaces. Our results are more general than metric versions of these type mappings.

Keywords: Modular metric spaces; fixed point; Hausdorff metric.

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1 Introduction

Let (X, d) be a metric space. A mapping $T : X \to X$ is a contraction if

$$d(T(x), T(y)) \leq k.d(x, y),$$

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for all $x, y \in X$, where $k < 1$.The Banach Contraction Mapping Principle appear in explict form in Banach's thesis in 1922[1]. Because of the simplicity and usefulness of this theory, it has become a very popular tool in solving existence problems in many fields of mathematical analysis. So a number of authors have extended and generalized Banach's Contraction Principle in many different directions. Some of these authors such as Rakotch[2], Boyd and Wong [3], Rhoades[4] investigated weaker contrative conditions using a control function $\alpha : [0,\infty) \to [0,1)$ in place of the contraction constant $k \in (0,1)$. After Rhoades, several number of results appeared in fixed point theory. These results could be seen in the papers of Zang and Song [5], Doric [6], Hosseini [7] etc. In 1969 Nadler [8] gave fixed point results for multivalued mappings in metric spaces.

The notion of modular space was given by Nakano [9] and it was intensively developed by Musielak and Orlicz, Koshi and Shimogaki, Yamamuro(see [10, 11, 12]) and others. A lot of mathematicians have been interested in fixed point theory in modular spaces . In 2006, Chistyakov introduced the notion of metric modulars inspired by the classical linear modulars [13] and in 2008 he gave concpet of modular metric spaces generated by F-modulars [14]. In 2010 Chistyakov gave the notion of modular metric spaces and properties of these spaces [15] and he gave a fixed point theorem for these spaces in 2011 [16]. After Chistyakov's results, a lot of authors interested fixed point results in modular metric spaces [17, 18, 16, 19, 20, 21, 22]. Kılınç and Alaca [23] defined $(\varepsilon, k)-$uniformly locally contractive mappings and η-chainable concept and proved a fixed point theorem for these concepts in a complete modular metric spaces. Kılınç and Alaca [24] proved two main fixed point theorems for commuting mappings in modular metric spaces. In 2014 Khamsi and Abdou investigated Hausdorff metric modular in modular metric spaces and proved fixed point theorem for multi-valued mappings [25]. Chaipunya et. al. gave some fixed point results of multivalued mappings in modular meric spaces [26].Tukoglu and Kılınç [27]. gave modular metric versions of Khojasteh, Karapinar and Khandani's results.

In this paper we investigated fixed point results for multivalued mappings which satisfies $(\varphi, \psi)-$contraction condition in complete modular metric spaces.

1.1 Preliminaries

In this section, we will give some basic concepts and definitions about modular metric spaces which are useful for our results.

Definition 2.1 [[15], Definition 2.1] Let X be a nonempty set, a function $w : (0,\infty) \times X \times X \to [0,\infty]$ is said to be a metric modular on X if satisfying, for all $x, y, z \in X$ the following condition holds:

(i) $w_\lambda(x, y) = 0$ for all $\lambda > 0 \Leftrightarrow x = y$;

(ii) $w_\lambda(x, y) = w_\lambda(y, x)$ for all $\lambda > 0$;

(iii) $w_{\lambda+\mu}(x, y) \le w_\lambda(x, z) + w_\mu(z, y)$ for all $\lambda, \mu > 0$.

If instead of (i), we have only the condition

$(\acute{i})$ $w_\lambda(x, x) = 0$ for all $\lambda > 0$, then w is said to be a (metric) pseudomodular on X.

The main property of a metric modular [15] w on a set X is the following: given $x, y \in X$, the function $0 < \lambda \mapsto w_\lambda(x, y) \in [0,\infty]$ is nonincreasing on $(0,\infty)$. In fact, if $0 < \mu < \lambda$, then (iii), $(\acute{i})$ and (ii) imply

$$w_\lambda(x, y) \le w_{\lambda-\mu}(x, x) + w_\mu(x, y) = w_\mu(x, y).$$

It follows that at each point $\lambda > 0$ the right limit $w_{\lambda+0}(x, y) = \lim_{\mu\to\lambda+0} w_\mu(x, y)$ and the left limit $w_{\lambda-0}(x, y) = \lim_{\varepsilon\to+0} w_{\lambda-\varepsilon}(x, y)$ exist in $[0,\infty]$ and the following two inequalities hold:

$$w_{\lambda+0}(x, y) \le w_\lambda(x, y) \le w_{\lambda-0}(x, y).$$

Theorem 2.1 [21] Let X_w be a complete modular metric space and T a contraction on X_w. Then, the sequence $(T^n x)_{n\in\mathbb{N}}$ converges to the unique fixed point of T in X_w for any initial $x \in X_w$.

Now we give some definitions, which are useful for our main results.

Definition 2.2 Let X_w be a modular metric space. Then following definitions exist:

(1) The sequence $(x_n)_{n\in\mathbb{N}}$ in X_w is said to be convergent to $x \in X_w$ if $w_1(x_n, x) \to 0$, as $n \to \infty$

(2) The sequence $(x_n)_{n\in\mathbb{N}}$ in X_w is said to be Cauchy if $w_1(x_m, x_n) \to 0$, as $m, n \to \infty$

(3) A subset C of X_w is said to be closed if the limit of a convergent sequence of C always belong to C.

(4) A subset C of X_w is said to be complete if any Cauchy sequence in C is a convergent sequence and its limit is in C.

(5) A subset C of X_w is said to be $w-$bounded if

$$\delta_w(C) = \sup\{w_1(x,y); x, y \in C\} < \infty.$$

(6) A subset C of X_w is said to be $w-$compact if for any (x_n) in C there exists a subset sequence (x_{n_k}) and $x \in C$ such that $w_1(x_{n_k}, x) \to 0$

(7) w is said to satisfy the Fatou property if and only if for any sequence $(x_n)_{n\in\mathbb{N}}$ in X_w $w-$convergent to x, we have

$$w_1(x,y) \leq \liminf_{n\to\infty} w_1(x_n, y),$$

for any $y \in X_w$.

Now we will give some basic properties and notions of multivalued mappings in modular metric spaces which was given in [25] For a subset M of modular metric space X_w set

(i) $\mathcal{CB}(M) = \{C : C \text{ is nonempty } w-\text{closed and } w-\text{bounded subset of } M\}$;

(ii) $K(M) = \{C : C \text{ is nonempty } w-\text{compact subset of } M\}$;

(iii) the Haussdorf modular metric is defined on$\mathcal{CB}(M)$ by

$$H_w(A,B) = \max\left\{\sup_{x\in A} w_1(x,B), \sup_{y\in B} w_1(y,A)\right\},$$

where $w_1(x,B) = \inf_{y\in B} w_1(x,y)$.

Definition 2.3 Let X_w be a complete modular metric space and M be a nonempty subset of X_w. A mapping $T : M \to \mathcal{CB}(M)$ is called a multivalued Lipschitzian mapping, if there exists a constant $k > 0$ such that

$$H_w(Tx, Ty) \leq k w_1(x,y),$$

for any $x, y \in M$.

A point $x \in M$ is called fixed point of T whenever $x \in Tx$.The set of fixed points of T will be denoted by Fix(T)

It was shown in[25] that Definition 2.3 is more general than Theorem 2.1.

Definition 2.4 A function $\psi : [0,\infty] \to [0,\infty)$ is called an *altering distance function* if it satisfies the following conditions:

(1) ψ is monotone increasing and continuous; (2) $\psi(t) = 0$ if and only if $t = 0$

2 Main Results

In this section we will give a fixed point theorem for multivalued mappings which satisfies $(\varphi, \psi)-$ contraction condition.

Theorem 2.1. *Let X_w be a complete modular metric space, $\emptyset \neq M \subseteq X_w$ and $T : M \to K(M)$ be a multivalued mapping satisfies following conditions with Fatou Property:*

$$\psi(H_w(Tx, Ty)) \leq \psi(M_w(x, y)) - \varphi(N_w(x, y)) \tag{3.1}$$

for all $x, y \in M$ and $x \neq y$; where

$$\begin{aligned} M_w(x,y) &= \max\left\{w_1(x,y), \delta_1(x,Tx), \delta_1(y,Ty), \frac{1}{2}(\delta_2(x,Ty) + \delta_2(y,Tx))\right\} \\ N_w(x,y) &= \min\left\{w_1(x,y), \delta_1(x,Tx), \delta_1(y,Ty), \frac{1}{2}(\delta_2(x,Ty) + \delta_2(y,Tx))\right\} \end{aligned}$$

Let $\varphi(t) : [0, \infty] \to (0, \infty)$, $\varphi(t) > 0$ is semicontinuous for all $t > 0$ and discontinuous at $t = 0$;
$\psi(t) : [0, \infty] \to [0, \infty)$ be altering distance function.

Then T has a fixed point in $M \subseteq X_w$; where $w_1(x_0, x_1) < \infty$ for some x_0, $x_1 \in X_w$ and $H_w(A, B)$ is modular Hausdorff metric.

Proof. Let $x_0 \in M$ be arbitrary and $x_1 \in Tx_0$. Then there is $x_2 \in Tx_1$ such that

$$w_1(x_1, x_2) \leq H_w(Tx_0, Tx_1)$$

Since ψ is monotone increasing we get

$$\psi(w_1(x_1, x_2)) \leq \psi(H_w(Tx_0, Tx_1))$$

If we apply (3.1), we get

$$\psi(w_1(x_1, x_2)) \leq \psi(H_w(Tx_0, Tx_1)) \leq \psi(M_w(x_0, x_1)) - \varphi(N_w(x_0, x_1)) \tag{3.2}$$

When we write x_{2n} instead of x_0, x_{2n+1} instead of x_1 and x_{2n+2} instead of x_2 in (2.2) and expanded the inequality we get

$$\psi(w_1(x_{2n+1}, x_{2n+2})) \leq \psi(H_w(Tx_{2n}, Tx_{2n+1})) \leq \psi(M_w(x_{2n}, x_{2+n1})) - \varphi(N_w(x_{2n}, x_{2n+1})) \tag{3.3}$$

where

$$\begin{aligned} M_w(x_{2n}, x_{2n+1}) &= \max\left\{ \begin{array}{c} w_1(x_{2n}, x_{2n+1}), \delta_1(x_{2n}, Tx_{2n}), \delta_1(x_{2n+1}, Tx_{2n+1}), \\ \frac{1}{2}(\delta_2(x_{2n}, Tx_{2n+1}) + \delta_2(x_{2n+1}, Tx_{2n})) \end{array} \right\} \\ \delta_1(x_{2n}, Tx_{2n}) &= \inf_{x_{2n+1} \in Tx_{2n}} \{w_1(x_{2n}, x_{2n+1})\} \leq w_1(x_{2n}, x_{2n+1}) \\ \delta_1(x_{2n+1}, Tx_{2n+1}) &= \inf_{x_{2n+2} \in Tx_{2n+1}} \{w_1(x_{2n+1}, x_{2n+2})\} \leq w_1(x_{2n+1}, x_{2n+2}) \\ \delta_2(x_{2n}, Tx_{2n+1}) &= \inf_{x_{2n+2} \in Tx_{2n+1}} \{w_1(x_{2n}, x_{2n+2})\} \\ \delta_2(x_{2n+1}, Tx_{2n}) &= \inf_{x_{2n+1} \in Tx_{2n}} \{w_1(x_{2n+1}, x_{2n+1})\} = 0 \end{aligned}$$

is satisfied and (2.3) is equal to

$$M_w(x_{2n}, x_{2n+1}) = \max\left\{w_1(x_{2n}, x_{2n+1}), w_1(x_{2n+1}, x_{2n+2}), \frac{1}{2}(w_2(x_{2n}, x_{2n+2}))\right\}$$

$$M_w(x_{2n}, x_{2n+1}) = \max\left\{w_1(x_{2n}, x_{2n+1}), w_1(x_{2n+1}, x_{2n+2}), \frac{1}{2}(w_1(x_{2n}, x_{2n+1}) + w_1(x_{2n+1}, x_{2n+2}))\right\}$$

Let us assume that

$$M_w(x_{2n}, x_{2n+1}) = w_1(x_{2n+1}, x_{2n+2})$$

If we consider this assumption in equation (3.3) we conclude that

$$\psi(w_1(x_{2n+1}, x_{2n+2})) \leq \psi(w_1(x_{2n+1}, x_{2n+2})) - \varphi(N_w(x_{2n}, x_{2n+1})) \tag{3.4}$$

Since $\varphi > 0$

$$\psi(w_1(x_{2n+1}, x_{2n+2})) < \psi(w_1(x_{2n+1}, x_{2n+2})) \tag{3.5}$$

Since ψ is monotone increasing we get

$$w_1(x_{2n+1}, x_{2n+2}) < w_1(x_{2n+1}, x_{2n+2}) \tag{3.6}$$

But this is a contradiction . Then we conclude that

$$M_w(x_{2n}, x_{2n+1}) = w_1(x_{2n}, x_{2n+1})$$

Thus $(w_1(x_{2n}, x_{2n+1}))$ is a monotone decreasing sequance. Since $K(M)$ is compact, it is closed and bounded and it is bounded from above. That is for $r > 0$, we get

$$\lim_{n\to\infty} w_1(x_{2n}, x_{2n+1}) = r$$

If we take the limit for $n \to \infty$ in (3.3), we get

$$\begin{aligned}
\lim_{n\to\infty} \psi(w_1(x_{2n+1}, x_{2n+2})) &\leq \lim_{n\to\infty} \psi(M_w(x_{2n}, x_{2+n1})) - \lim_{n\to\infty} \varphi(N_w(x_{2n}, x_{2n+1})) \\
\lim_{n\to\infty} \psi(w_1(x_{2n+1}, x_{2n+2})) &\leq \lim_{n\to\infty} \psi(w_1(x_{2n}, x_{2+n1})) - \lim_{n\to\infty} \varphi(N_w(x_{2n}, x_{2n+1}))
\end{aligned}$$

for all $r > 0$. Since ψ is continuous

$$\begin{aligned}
\psi(\lim_{n\to\infty} w_1(x_{2n+1}, x_{2n+2})) &\leq \psi(\lim_{n\to\infty} w_1(x_{2n}, x_{2+n1})) - \lim_{n\to\infty} \varphi(N_w(x_{2n}, x_{2n+1})) \\
\psi(r) &\leq \psi(r) - \lim_{n\to\infty} \varphi(N_w(x_{2n}, x_{2n+1}))
\end{aligned}$$

From the definition of φ, $\varphi(N_w(x_{2n}, x_{2n+1})) \neq 0$. So we get

$$\psi(r) < \psi(r)$$

But this is a contradiction. Thus we get $r = 0$. That is

$$\lim_{n\to\infty} w_1(x_{2n}, x_{2n+1}) = 0$$

Now let us show that (x_n) is a Cauchy sequence. To show that is sufficiant to show that the subsequence (x_{2n}) is a Cauchy sequence. Assume that (x_{2n}) is not Cauchy, then there is

$$w_1(x_{2n}, x_{2m}) \geq \varepsilon$$

for $\exists\varepsilon > 0; m > n > n_0(\varepsilon)$ and m and n are the first numbers that satisfies the inequality above. We can write

$$\begin{aligned}
\psi(w_1(x_{2n}, x_{2m})) &\leq \psi(H_w(Tx_{2n-1}, Tx_{2m-1})) \\
&\leq \psi(M_w(x_{2n-1}, x_{2m-1})) - \varphi(N_w(x_{2n-1}, x_{2m-1}))
\end{aligned}$$

for $x_{2m} \in T(x_{2m-1})$ and $x_{2n} \in T(x_{2n-1})$.

$$\begin{aligned}
M_w(x_{2n-1}, x_{2m-1}) &= \max\left\{ \begin{array}{c} w_1(x_{2m-1}, x_{2n-1}), \delta_1(x_{2n-1}, Tx_{2n-1}), \delta_1(x_{2m-1}, Tx_{2m-1}), \\ \frac{1}{2}(\delta_2(x_{2n-1}, Tx_{2m-1}) + \delta_2(x_{2m-1}, Tx_{2n-1})) \end{array} \right\} \\
&= \max\left\{ \begin{array}{c} w_1(x_{2m-1}, x_{2n-1}), w_1(x_{2n-1}, x_{2n}), w_1(x_{2m-1}, x_{2m}), \\ \frac{1}{2}(w_2(x_{2n-1}, x_{2m}) + w_2(x_{2m-1}, x_{2n})) \end{array} \right\}
\end{aligned}$$

$$w_1(x_{2m-1}, x_{2n-1}) \leq w_{\frac{1}{2}}(x_{2m-1}, x_{2n}) + w_{\frac{1}{2}}(x_{2n}, x_{2n-1})$$

Since metric modular is monotone decreasing, there is $2\varepsilon > 0$ for $\frac{1}{2} > 0$ such that

$$w_{\frac{1}{2}}(x_{2m-1}, x_{2n}) < 2\varepsilon$$

Now if we take the limit for $m, n \to \infty$ we get

$$w_1(x_{2m-1}, x_{2n-1}) \leq 2\varepsilon$$

$$\begin{aligned} w_2(x_{2n-1}, x_{2m}) &\leq w_1(x_{2n-1}, x_{2n}) + w_1(x_{2n}, x_{2m}) \\ &\leq w_1(x_{2n}, x_{2m}) \leq w_{\frac{1}{2}}(x_{2n}, x_{2m-1}) + w_{\frac{1}{2}}(x_{2m-1}, x_{2m}) \\ w_2(x_{2n-1}, x_{2m}) &\leq 2\varepsilon \end{aligned}$$

From the main property of metric modular we can write the inequality below;

$$w_2(x_{2m-1}, x_{2n}) \leq w_1(x_{2m-1}, x_{2n}) < \varepsilon$$

When we write these inequalities we get

$$M_w(x_{2m-1}, x_{2n-1}) = \max\left\{2\varepsilon, 0, 0, \frac{1}{2}(2\varepsilon + \varepsilon)\right\} = 2\varepsilon$$

And if we write these results in (3.1) we get

$$\psi(2\varepsilon) \leq \psi(2\varepsilon) - \varphi(N_w(x_{2n-1}, x_{2m-1}))$$

Since $\varphi(N_w(x_{2n-1}, x_{2m-1}))$ can't equal to zero this inequality turns into

$$\psi(2\varepsilon) < \psi(2\varepsilon)$$

which is a contradiction. Hence our assumption is wrong and (x_{2n}) is a Cauchy sequence.

Now let us show the existence of the fixed point. Let us assume otherwise, that is $\bar{x}$ is not a fixed point of T,while (x_n) $\to \bar{x}$.

Since K(M) is compact, it is also closed and bounded. So there is a $\bar{x}$ $\in$K(M)$\subseteq$ X_w such that (x_n) $\to \bar{x}$. Then from the Fatou property we get

$$\delta_1(\bar{x}, T\bar{x}) \leq \lim_{n\to\infty} \inf w_1(x_{n+1}, T\bar{x}) = \lim_{n\to\infty} \inf w_1(Tx_n, T\bar{x}) \leq \lim_{n\to\infty} H_w(Tx_n, T\bar{x})$$

Hence we can write that

$$\begin{aligned} \psi(\delta_1(\bar{x}, T\bar{x})) &\leq \psi(H_w(Tx_n, T\bar{x})) \\ &\leq \psi(M_w(x_n, \bar{x}) - \varphi(N_w(x_n, \bar{x}) \end{aligned}$$

$$\begin{aligned} M_w(x_n, \bar{x}) &= \max\left\{w_1(x_n, \bar{x}), \delta_1(x_n, Tx_n), \delta_1(\bar{x}, T\bar{x}), \frac{1}{2}(\delta_2(x_n, \bar{x}) + \delta_2(\bar{x}, Tx))\right\} \\ \lim_{n\to\infty} M_w(x_n, \bar{x}) &= \max\left\{w_1(\bar{x}, \bar{x}), \delta_1(\bar{x}, T\bar{x}), \delta_1(\bar{x}, T\bar{x}), \frac{1}{2}(\delta_2(\bar{x}, T\bar{x}) + \delta_2(\bar{x}, \bar{x}))\right\} \\ &= \max\left\{0, 0, \delta_1(\bar{x}, T\bar{x}), \frac{1}{2}(\delta_2(\bar{x}, T\bar{x}))\right\} \end{aligned}$$

From the main property of a modular metric we can write

$$\delta_2(\bar{x}, T\bar{x}) \leq \delta_1(\bar{x}, T\bar{x})$$

Hence we get

$$\lim_{n\to\infty} M_w(x_n, \bar{x}) = \delta_1(\bar{x}, T\bar{x})$$

From this equality we find that

$$\psi(\delta_1(\bar{x}, T\bar{x})) \leq \psi(\delta_1(\bar{x}, T\bar{x})) - \varphi(N_w(x_n, \bar{x})$$

From the definition of φ this inequality turns into

$$\psi(\delta_1(\bar{x}, T\bar{x})) < \psi(\delta_1(\bar{x}, T\bar{x}))$$

which is a contradiction. Hence our assumption is wrong and $\bar{x}$ is a fixed point of $T\bar{x}$. That is $\bar{x} \in T\bar{x}$. This completes the proof. □

3 Conclusions

In this paper we give $\varphi - \psi$ contraction for multivalued mappings in modular metric spaces and a fixed point result is shown. We try to expand fixed point theory for modular metric spaces. Authors can develop this results for other spaces like generalised modular metric spaces. Also authors can see [16],[15],[25] and references therein for more information about modular metric spaces.

Competing Interests

Authors have declared that no competing interests exist.

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