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Modeling and Parameter Sensitivity Analysis of Population Dynamics in Uganda

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Author's contribution

The author JO conceived the idea, modeling and analysis, read and approved the manuscript.

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Abstract

Keeping track of the human population is essential for proper planning for facilities such as healthcare, infrastructure, education, and other essential needs. There are various ways by which the government can ensure that service provision is improved and maintained for its citizens and very often this starts by knowing the changes in demography as a function of time.In this work mathematical modeling and simulations are used to study the population dynamics of Uganda. The models are used for prediction of the county's population and how its dynamics changes in time. Parameter sensitivity analysis was performed using population census data and the results shows huge influence of variations of the model parameters. The results show that the difference between the per capita birth and death rate parameters is crucial for changes in the country's population. Such findings can also be analogously applied to countries with a similar population structure and economy.

Keywords: *Population growth; parameter sensitivity analysis; modeling; birth rate; death rate.*

1 Introduction

Currently Uganda has one of the highest population growth rates in the world [1,2]. Already the provision of essential social (and basic and needs) services such as water, housing, education, hospitals is a challenge to not only the developed countries but even more so to the developing countries such as Uganda. It is often poorly understood how changes in certain crucial parameters that influence the total population of people in a country. Gaining insight into how small and large variations in parameters impact on the national population is useful not only for the government planning but also for ensuring that in the event of sudden outbreak of epidemics, the communities are well prepared to cope.

One of the tools that have been effectively used in tacking such challenges is mathematical modeling and simulation. However, this process requires good quality data that is collected from population census. Such data acquisition processes are very expensive and time-consuming. Many

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third world countries are not equipped both financially and technologically to cope with such challenges alongside the already existing ones. Therefore, by using existing data from previous studies or information available from government archives, mathematical modeling and simulations can be used to make predictions of population in the future, and the associated implications of such predictions on the daily lives of the people (as demonstrated and discussed in [3]). We live in a universe of limited resources and having a population that nearly doubles within every two decades puts the country on a trajectory of unsustainable. With the ever-increasing demand for food, water and fuels, it has become very crucial for every community to plan for its population. Proper planning for the population will ensure that the limited resources are not depleted, but rather sustained within the next few decades and beyond.

In this paper mathematical modeling and simulations are performed using prior knowledge on parameters that are considered crucial for understanding the population dynamics of Uganda. In literature, little attention has been devoted to unraveling the complexity in the dynamics of population growth models for a third world country such as Uganda, e.g. [4]and yet in a country like Uganda adequate provision of many basic needs such as shelter, water and other socio-economic needs lags behind. There is also a problem of insufficient communal facilities such as public toilets and or latrines for hygiene, good sanitation, infrastructure and general welfare (see [5] for detailed insight). Adequate provision for most if not all these elements relies heavily on the government having up to date knowledge – not only on the number of people living on the country but also those that immigrate, emigrate and or die. First, an analytic assessment of the model is given. This is then followed by an assessment of the implications of the results from the computer simulation. Already, the country's population has nearly doubled in just about two decades. Such a rapid population growth can be argued to be essential for the country's economic growth. It still remains a challenge for Uganda to cope with its high birth rate which by the year 2006 was reported to be at about 3.1% with a total birth rate of about 6 to 7 children per woman.

Recently an article published in the New Vision newspaper Uganda on December 14th, 2012 highlighted the potential benefits and challenges that might arise from the young and rapidly growing population of Uganda. They noted that a total of 78% of Ugandans were below the age of 30 years and 52% were below the age of 15 years. They also observed that 6.5 million Ugandans in the age group 18-30 years constituted 21.3% of the national population – an age group that is projected to grow to about 7.7 million by 2015. Clearly, this kind of demography is worrying and a lot remains to be done by the relevant government authorities charged with planning, allocation and management of national resources. It would be naïve to assume that such demography and associated problems is limited to Uganda. The findings and recommendations from this study would therefore be considered applicable to countries with similar demography and challenges as that of Uganda, e.g. Kenya, Burkina Faso, Ethiopia, Zambia, Zimbabwe and Niger among others.

2. Methodology

2.1 Population Growth Models

There are many models that have been proposed in literature as a state-of-the-art approach for modeling population growth. These models include: the Exponential population growth model by Malthus [6]; the Logistic growth model (e.g. [7]) – which is a modified version of Malthus' model, the Gompertz population growth model (e.g. [8]). Details of the theory and underlying principles are not discussed here since they can be found in literature. Many variants of these models have been very useful in making predictions of various populations of species e.g. Fish, human and even microbial populations. These predictions are necessary since they can be used for monitoring populations and ensuring that specific species of animals already well protected for instance in national parks or large water reserves where fishing is not allowed. However, the need to use such mathematical models to make predictions of population growth is essential for every society and nation – particularly for those with rapidly changing demography in third world countries e.g. Uganda. In this work, an illustration of how mathematical models can be used to identify sensitive parameters is demonstrated.

The logistic population growth model has for long been used for making prediction and studying the population dynamics of different specifies of animals (see e.g. [7][9]. In some cases they are applied to a specific population. Consider the model for the population growth as described by Wali et al. [4] which is given as:

$$\frac{d}{dt}P(t) = \alpha P(t)\left(1 - \frac{\beta}{\alpha}P(t)\right); \;\; P(0) > 0 \tag{1}$$

where $P(t)$ is the population in the country at any given point in time t. Here α and β are referred to as the vital parameters of the population; α is often referred to as the Malthusian parameter, and it is basically the difference between the per-capita birth and death rate in a given population. The parameter β is the amount by which the difference in per-capita birth and death rate changes in response to the addition of one individual to the population. It is important to note that the logistic population growth model is a modification of the Malthusian model – which is in principle a simple exponential growth model.

The model in eq. (1) is much more realistic compared to the model proposed by Malthus [6] in which it was assumed that the population grows at a rate proportional to the size of the population of species under consideration. Malthus model assumption is only reasonable for a population living under ideal conditions of adequate nutrition, vast and unlimited environment in which the species lives, absence of any predators and disease among other things.

Let the population at some arbitrary defined starting time zero be $P(0)$. Let the parameters be denoted as $\vartheta_r \in \{\alpha, \beta\}$. The differential equation given in eq. (1) has the solution:

$$P(t) = \frac{\beta}{\alpha}\left(P(0)/\left(P(0) + \left(\frac{\beta}{\alpha} - P(0)\right)e^{-\alpha t}\right)\right) \tag{2}$$

This expression can be used to estimate the population of a given population at a specific time instant given that some information is available about the parameters and initial conditions. In the limiting time ($t \to \infty$), the population carrying capacity is obtained as $P(\infty) = \alpha/\beta$ for $\beta \neq 0$. At steady state $dP/dt = 0$, and from here-on the notation $\tilde{P}$ is used to denote the steady state population value. By setting the left hand side of eq. (1) to zero and solving for $\tilde{P}(0)$ on the right hand side, clearly, $\tilde{P}(0) \gg 0$the population at "time zero" is far from zero, in this case it is in millions of people), which implies that $\tilde{P} = \alpha/\beta$. Clearly, if $\alpha \gg \beta$ then the maximum population attained in the distant futurebecomes extremely large. This would imply too many people sharing limited resources. In the above model formulation the (per capita) birth and death parameters are coalesced together into a single parameter. In principle the value of this parameter can vary depending on changes in demographics and social economic factors in any given population.

2.2 Parameter Sensitivity Analysis

To study the sensitivity of the parameter in the population growth model, the expression in eq. (1), is differentiated partially with respect to each parameter in the model. Generally, global sensitivity analysis deals with the various outcomes the structure of the model of interest is capable of outputting. It explores all the considerably reasonable range of parameter. This kind of analysis is often done ad hoc with the help of the Metropolis-Hastings or Genetic Algorithms. However, since the model structure for the population growth is known (as given in eq. (1)), and the specific parameters of interest are also chosen, i.e. α and β; it is then preferable to use the local sensitivity analysis. Local sensitivity analysis basically refers to the sensitivity of parameters with respect to a given set of parameters.

First let us define the local sensitivity function in order to be able to analyze the model for the population growth. Generally the local sensitivity function takes the form:

$$dS_P(t,\vartheta_r)/dt = (\partial\phi/\partial P)S_P(t,\vartheta_r) + (\partial\phi/\partial\vartheta_r) \tag{3}$$

Let us define the sensitivity matrix by: $S_P(t,\vartheta_r) := \partial P(t)/\partial\vartheta_r \in \mathbb{R}^{N\times q}$ where N is the length of time instants in years for which the population is enumerated, q – number of parameters of interest. Values of $N = 1000$ and $q = 2$ are used in this paper.where ϕ represents the right hand side of eq. (1) and $\partial\phi/\partial P$ is a diagonal system Jacobian matrix. The theory and details of applications of sensitivity analysis can be found in the work of Gunawan et al. [10] and Keesman [11]. The sensitivity functions are obtained by computing the partial derivatives with respect to each of the parameters in eq. (1). The time-dependent parameter sensitivities are computed using the expressions given in eqs. (1) and (3) for each parameter. The resulting sensitivities are obtained by substituting the computed partial derivatives into the expression in eq. (3). This leads to the expressions:

$$dS_P(t,\alpha)/dt = (\alpha - 2\beta P)S_P(t,\alpha) + P \tag{4}$$

$$dS_P(t,\beta)/dt = (\alpha - 2\beta P)S_P(t,\beta) - P^2 \tag{5}$$

Since the initial values for the population does not depend on the parameters, then the initial conditions satisfythe condition $S_P(0,\alpha) = S_P(0,\beta) = 0$.The eqs. (4) and (5) are then solved to obtain the sensitivity functions$S_P(t,\alpha)$ and $S_P(t,\beta)$.

2.3 Evaluation of Parameter Sensitivities

Since the parameters and state variables of a given population model may take on a wide range of values, it is important to assess how these parameters influence the population.Here the parameter and state variables are $\vartheta_r \in \{\alpha,\beta\}$ and $P(t)$, respectively. The ranking metric is computed from the area under the parameter sensitivity function using:

$$\Phi_{\vartheta_r} = \int_{t(0)}^{t(f)} \|S_P(t,\vartheta_r)\|_1 dt \tag{6}$$

where $t(0)$ and $t(f)$ are the start and final time instants. This integral sensitivity metric is based on time integrals that are analogous to the criterion described in [12]. The metric specified in eq. (6) was used to compute the parameter sensitivities from the expressions given in eq. (4) and (5). All the analysis and simulation in this paper was performed in the MATLAB® Version 7.12.0 (R2011a) software under the Windows®7 environment.

3. Results and Discussion

In the simulation, we used nominal parameter values of $\hat{\alpha} = \bar{\alpha} \approx 0.0356$ and $\hat{\beta} = \bar{\beta} \approx 1.20569e - 10$, respectively. In this case the nominal values are considered to be equal to the estimated parameter values from [4]. The modeling and analysis in this work are based on the population data provided in Table 1 (Appendix). Modeling and simulations was performed using this data and the results are provided in Fig. 1. The sum of sensitivity functions as specified in eq. (6) were computed thereby providing information on how sensitive the change in population is likely to be as a result of a change in the parameters of interest. The simulations yielded the sum of sensitivity values$\Phi_{\alpha} \approx 8.061e + 12$ and $\Phi_{\beta} \approx 0$, for the population per-capita birth and death rates, respectively.In the simulation the point $t(0)$ is considered to be at 1980. It is important to note that the point of inflection, which was computed to be at $t^{*} = 88$ years (at the year 2068, see [4]; with a small change of notation). This means that in less than a century half of the resources that can support the population will have been consumed. The population is sensitive to a change in α (Fig. 1). This implies that a drastic or sudden variation in the difference between the birth and death rates is likely to result in a rapid population change (Fig. 1: C).

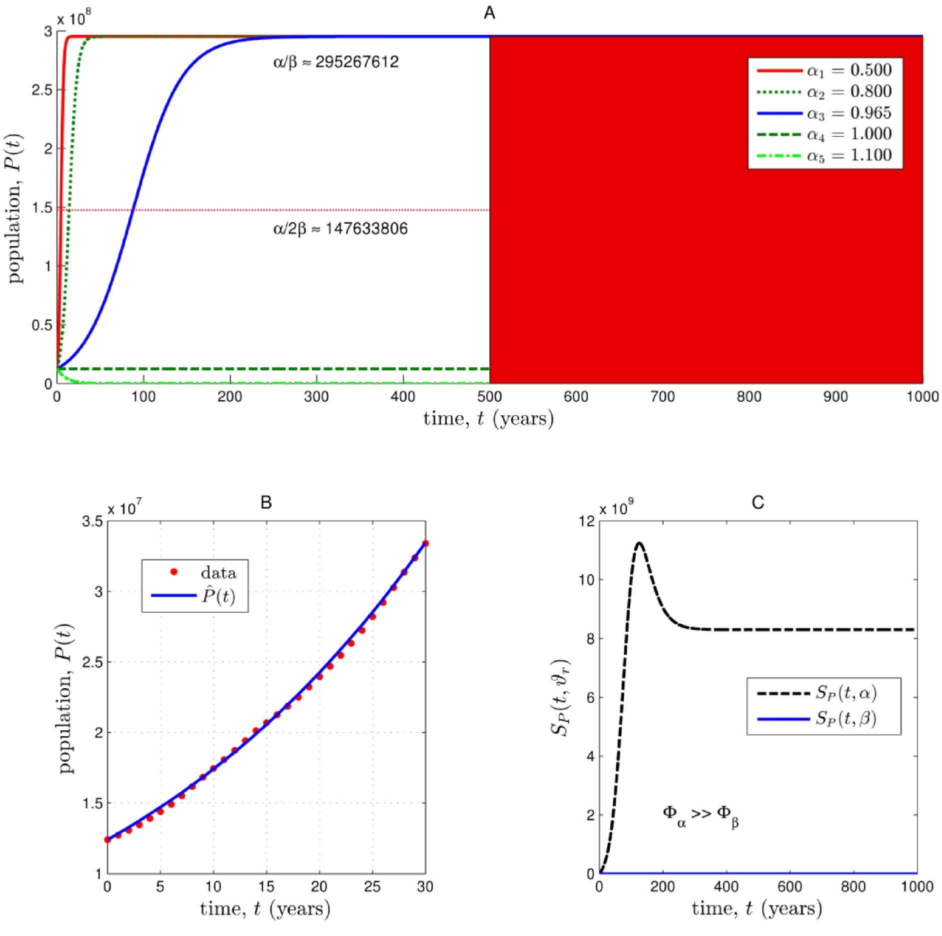

Fig. 1. Population growth and results of parameter sensitivity analysis.

A: projected population growth. The shaded region indicates the time frame at which the population carrying capacity if reached (assuming the population keeps growing). B: Plot showing the data values and predicted population in time, $\hat{P}(t)$. C: A plot of sensitivity function which shows that the change in sensitivities for α and β significantly differ with α being the more sensitive of the two.

As indicated in the results, assuming the population dynamics and demography continues in the projected path, then it is very likely that around in about 120 years into the future, i.e. counting from the year1980, the time for the $P(0)$, there is likely to be a significant influence on the demography. Off course these predictions are all based on the assumption that little will change in the rates of parameters such as immigration and emigration. This calls for perhaps even better models to that with the aid of demographic data in the future can help improve the predictions. This will aid improved planning for social and economic services for the population. Compared to decades ago, today Ugandan has a longer life expectancy and lower death rates (death as a result of all atrocities combined) – improved health care and education. This simply ensures that αincreases and so does the overall population.

Starting from time zero (t_0), we see that there is a rapid population growth within the next 100 years (Fig. 1A: blue line). A similar observation can also be made from Fig. 1C (first 100 years) from which there exists a rapid rise in value for the sensitivity function $S_P(t, \alpha)$. The trends from the figure indicate that there is need to pay particular attention to the alarmingly high population growth. Of course this no trivial task but both government and private institutions charged with educating the public on family planning and general welfare of women and children can help. A recent study has highlighted the gaps present in the provision of quality of family planning services in Uganda [13]. In their work they note that provision of quality family planning has long been overlooked in many parts of Uganda. Their study leads to recommendations such as the need to improve the quality of family planning services by ensuring that the limitations at the organizational and societal levels of society are adequately addressed.

The availability and affordability of birth control pills, availability of condoms and other birth control related health-care products still remains out of reach for most people. These issues were highlighted by the by World Health Organization [14]. This is a problem that likely to be further worsened by illiteracy and poverty for most people living in the rural areas. A reduced birth rate will slow down the rapid population growth, but it should also be noted that this will not automatically guarantee access to e.g. health care and education. They make achievement much more feasible. However, a reduction in the high growth rate (a reduced α) will surely impact positively on efforts to reduce poverty, provide better health-care, enhanced education to more people and a reduced burden on the environmental resources.

The models reveal that if the population per-capita growth rate remains at the current rate, then within the next ~500 years there is likely to be a heavy burden on the environmental resources. This projection is based on the assumption that no extreme catastrophe e.g. severe acute syndrome (SAS), Ebola and the emergence of new strains of the virus - human immune deficiency syndrome (HIV) AIDS or any other yet unknown factors that is likely to influence the population death rate. With rapid advances in technology in health-care, agriculture, biotechnology etc. the life expectancy of Ugandans has steadily risen in the past decade. The life expectancy as of 2012 was: 54.54 and 52.4 years for females and males, respectively; the life expectancy at birth for the total population: 53.45 years. These life expectancy values might not be as high as in developed countries, but is surely expected to further rise steadily. This rise is also likely to significantly influence the demography of Uganda within the next decade.

It is recommendable that for more accurate predictions the parameters α and β have to estimated perhaps every decade. This is because of variations that can arise from, e.g. inaccurate accountability of the number of people in the country Uganda, porous borders, non-registered and non-counted individuals and those entering the country illegally. All these are factors that can largely influence mathematical predictions, thereby, hugely biasing the resulting inferences and recommendations that arise from such analysis. One challenge that remains of interest to scientists is to understand the influx of people from rural into urban areas – thereby, putting a lot of pressure onto the already limited resources in the cities. This is a problem that is not only faced by Uganda but also other countries all over the world. It would be interesting to see more advancement into the development of mathematical models that help in the identification of sensitive (key) parameters that influence the drive for people to move from rural to urban areas.

The work in this paper already illustrates how hugely the parameter α affects the population in a given country – in this case Uganda.

4. Conclusion

The results in this work have demonstrated the impact of changes in parameters on the overall population of a community – as illustrated for the case of Uganda. A rapid in the per-capita birth growth rate parameters needs to be closely monitored by ensuring that quality demographic data is collected and used for better predictions and planning to improve the livelihoods of the citizens. The government and other authorities charged with service provision and education of the public especially on family planning and health education needs to upscale their efforts. In this work it has been shown that even by using a simple Logistic population growth model, the influence of how parameters influence the overall population in time can provide crucial insight into the population dynamics. The use of parameter sensitivity analysis in identifying crucial parameters and how they are likely to impact on the population growth has been demonstrated. Although not considered or discussed here, in real life there are surely other factors that influence on the population dynamics. Results from the mathematical modeling and simulations based on prior knowledge and demographic data is useful especially in cases when population growth needs to be accurately predicted for proper planning purposes. This work shows that the Logistic growth model is sufficiently powerful to accurately predict the population growth in time.

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Competing Interests

The author declares no competing interests.

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APPENDIX

Table 1. Actual values of population as specified in the manuscript by Wali et al. [4]

Year	Actual Population	Year	Actual Population	Year	Actual Population
1980	12414719	1991	18082137	2002	25469579
1981	12725252	1992	18729453	2003	26321962
1982	13078930	1993	19424376	2004	27233661
1983	13470393	1994	20127590	2005	28199390
1984	13919514	1995	20689516	2006	29206503
1985	14391743	1996	21248718	2007	30262610
1986	14910724	1997	21861011	2008	31367972
1987	15520093	1998	22502140	2009	32369558
1988	16176418	1999	23227669	2010	33398682
1989	16832384	2000	23955822		
1990	17455758	2001	24690002		

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